

Ecole polytechnique fédérale de Lausanne EPFL

School of Architecture, Civil and Environmental Engineering ENAC

ENAC - Environmental Science and Engineering Section ENAC-SSIE

Dynamic MFA Modelling Background and Examples

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Vita:

Physicist by training (Jena, 2007)

PhD in industrial ecology (NTNU Trondheim, Norway, 2013)

Professor for industrial ecology, Freiburg (since 2021)

Research:

Future material demand, circular economy potentials, and transformation pathways for society's metabolism, focus on the built environment: buildings, transport, infrastructure, energy supply; city to global level; major materials (steel, cement, wood, copper, plastics) and technology metals

Socio-metabolic inequality of resource use and access to stocks and flows in the built environment

Development of indicators, methods, models & data infrastructure



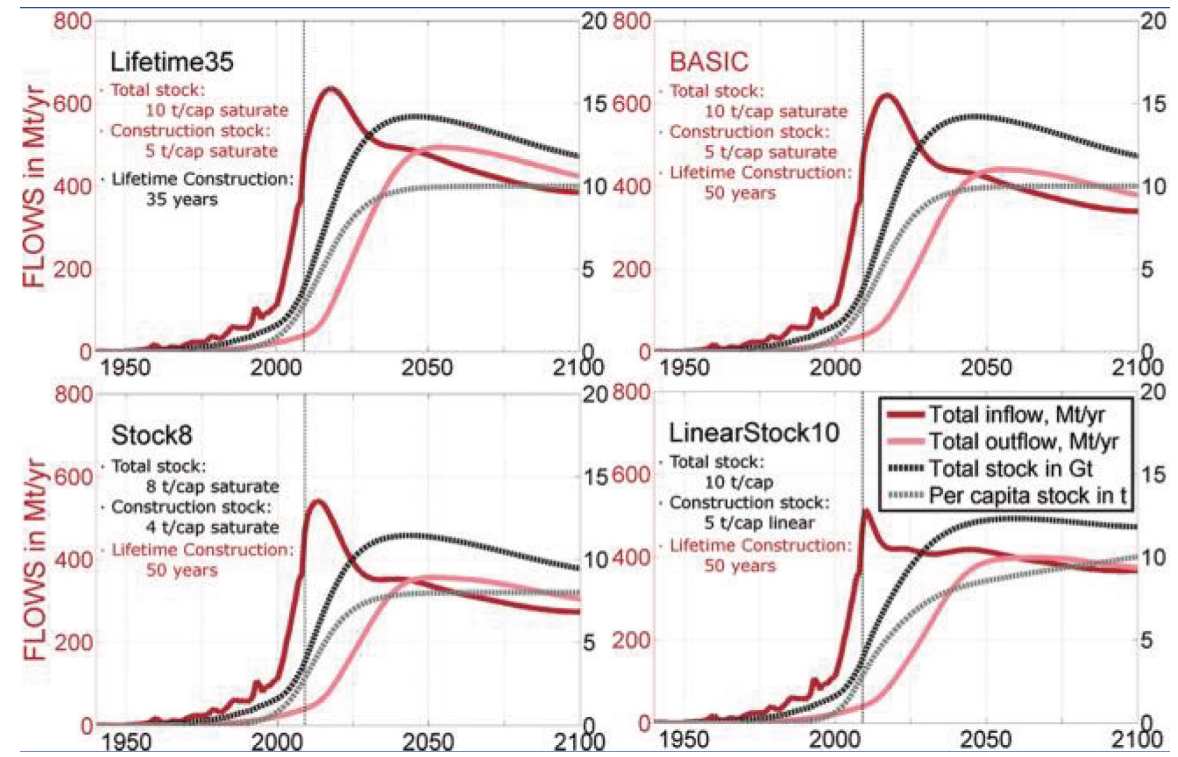
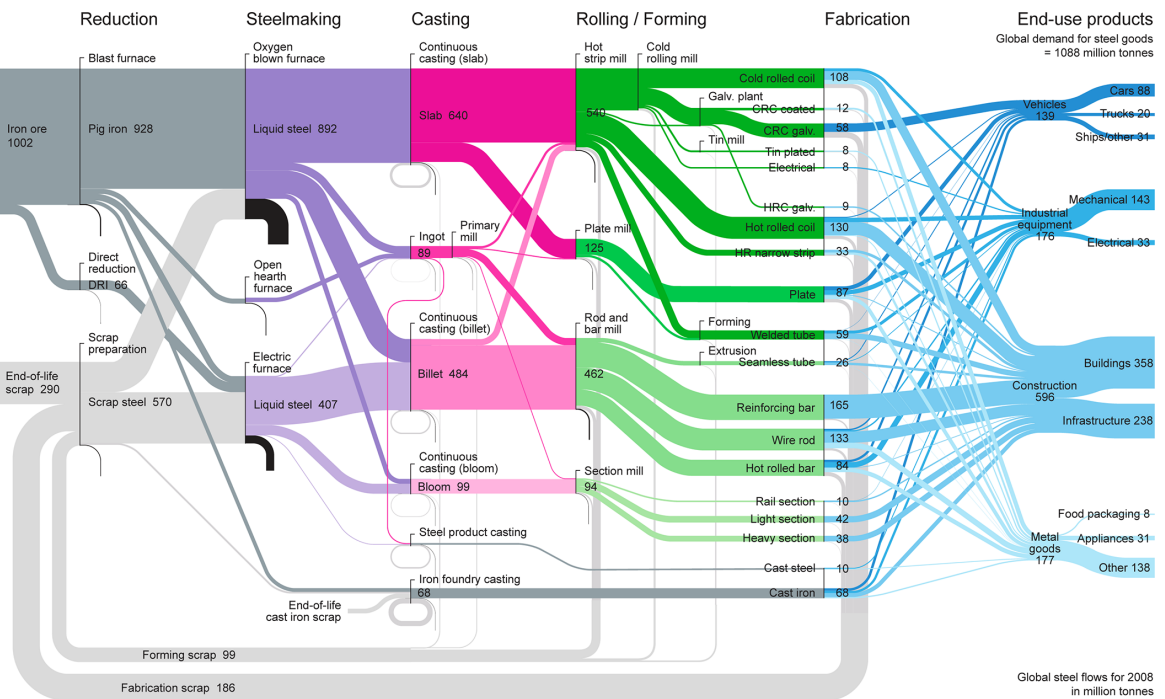
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Dynamic MFA

Introduction



Static MFA (=single-year snapshot) shows current status at high resolution, but no trends or transformation options into the future → need to trace material stocks and flows over time (= dynamic MFA / dMFA)



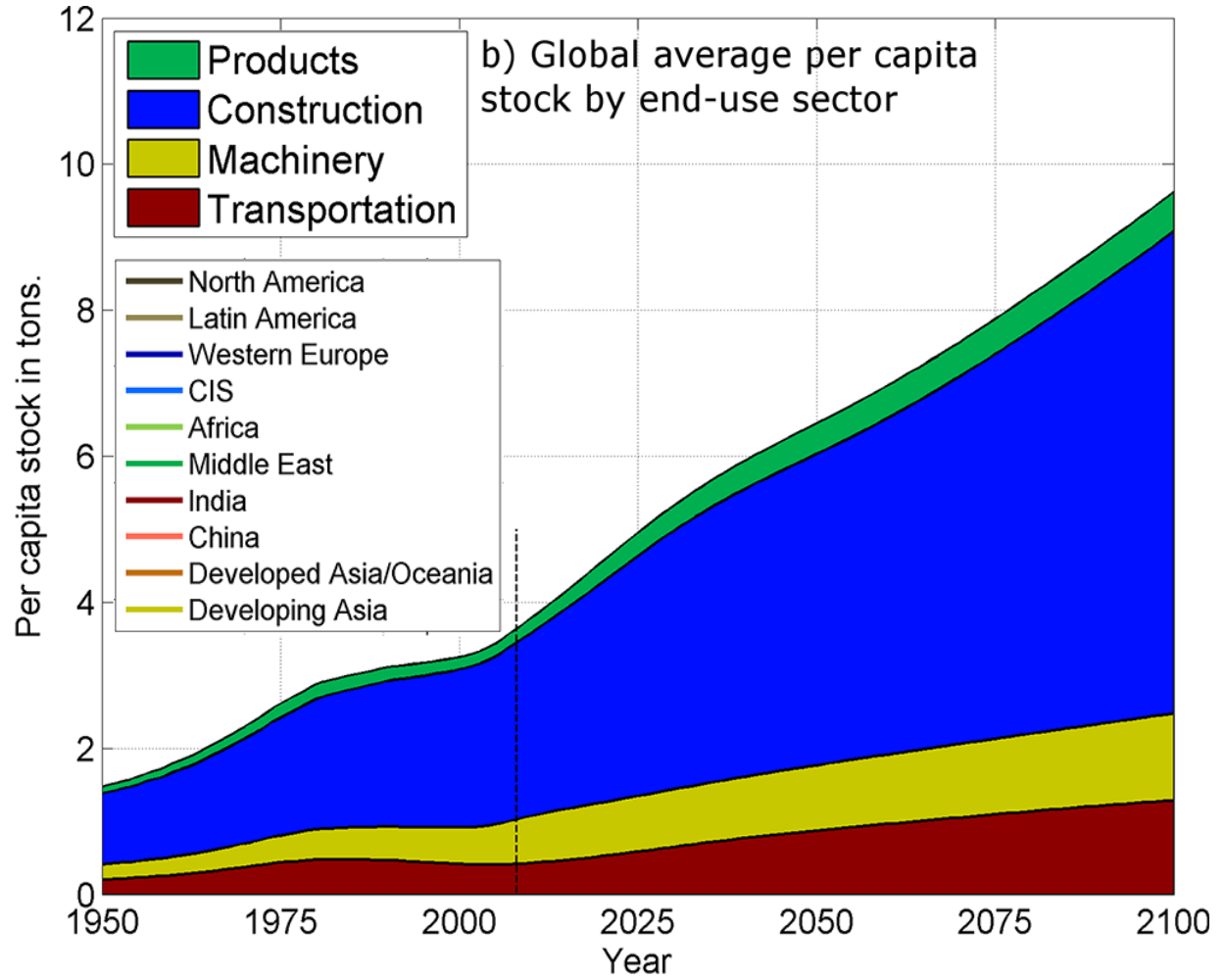
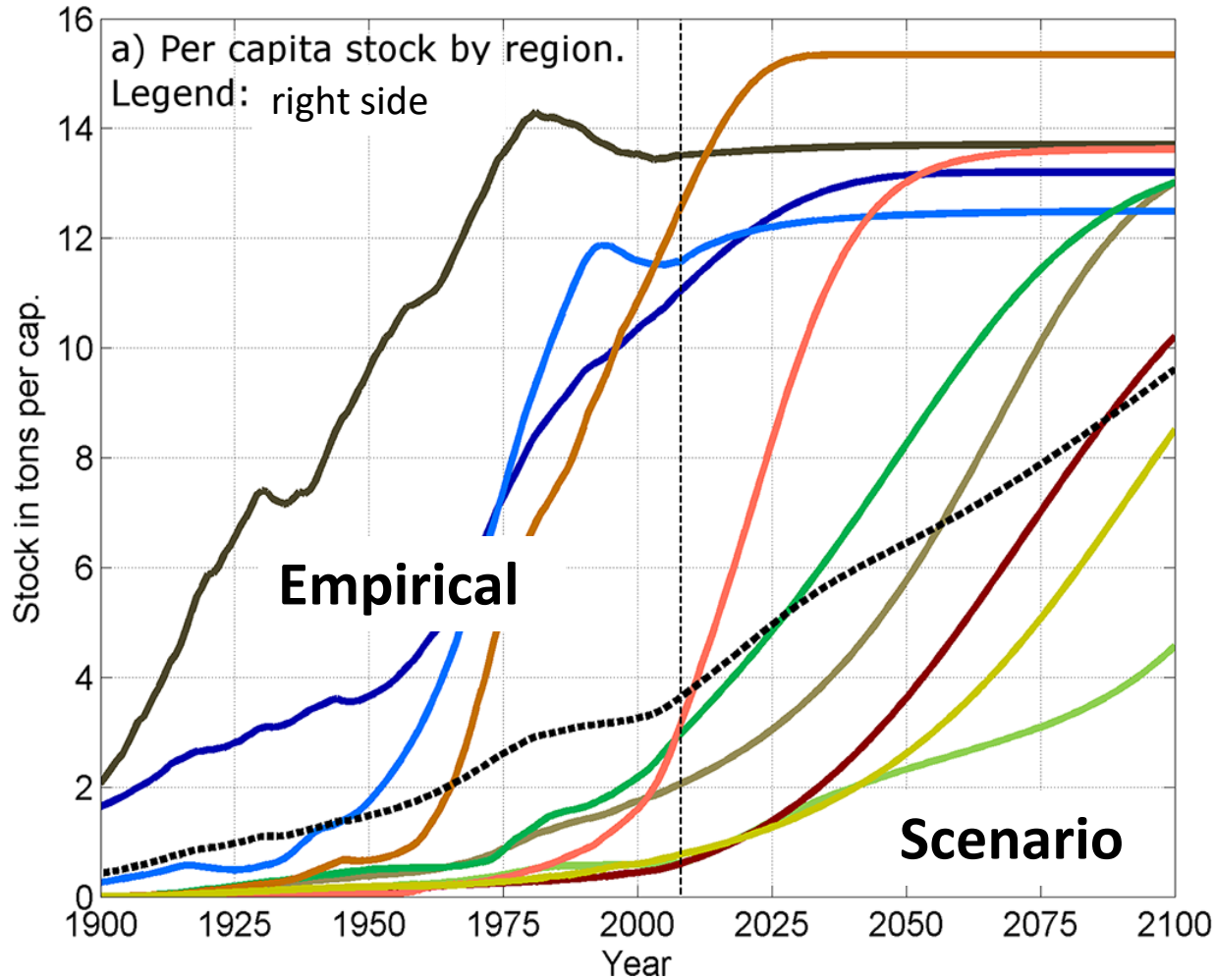
In dynamic material and energy flow analysis, we trace material and energy flows and stocks over time. We study how changing demand and new technology impact material and energy flows, how different decoupling strategies together can reach a certain sustainability target, and how the gradual shift of in-use stocks to new technologies leads to lower energy demand and supply of recyclable scrap from old products, vehicles, infrastructure, and buildings.

The global steel industry. From Cullen et al. (2012). <http://dx.doi.org/10.1021/es302433p>

Moving Toward the Circular Economy: The Role of Stocks in the Chinese Steel Cycle. From Pauliuk et al. (2012). <https://doi.org/10.1021/es201904c>



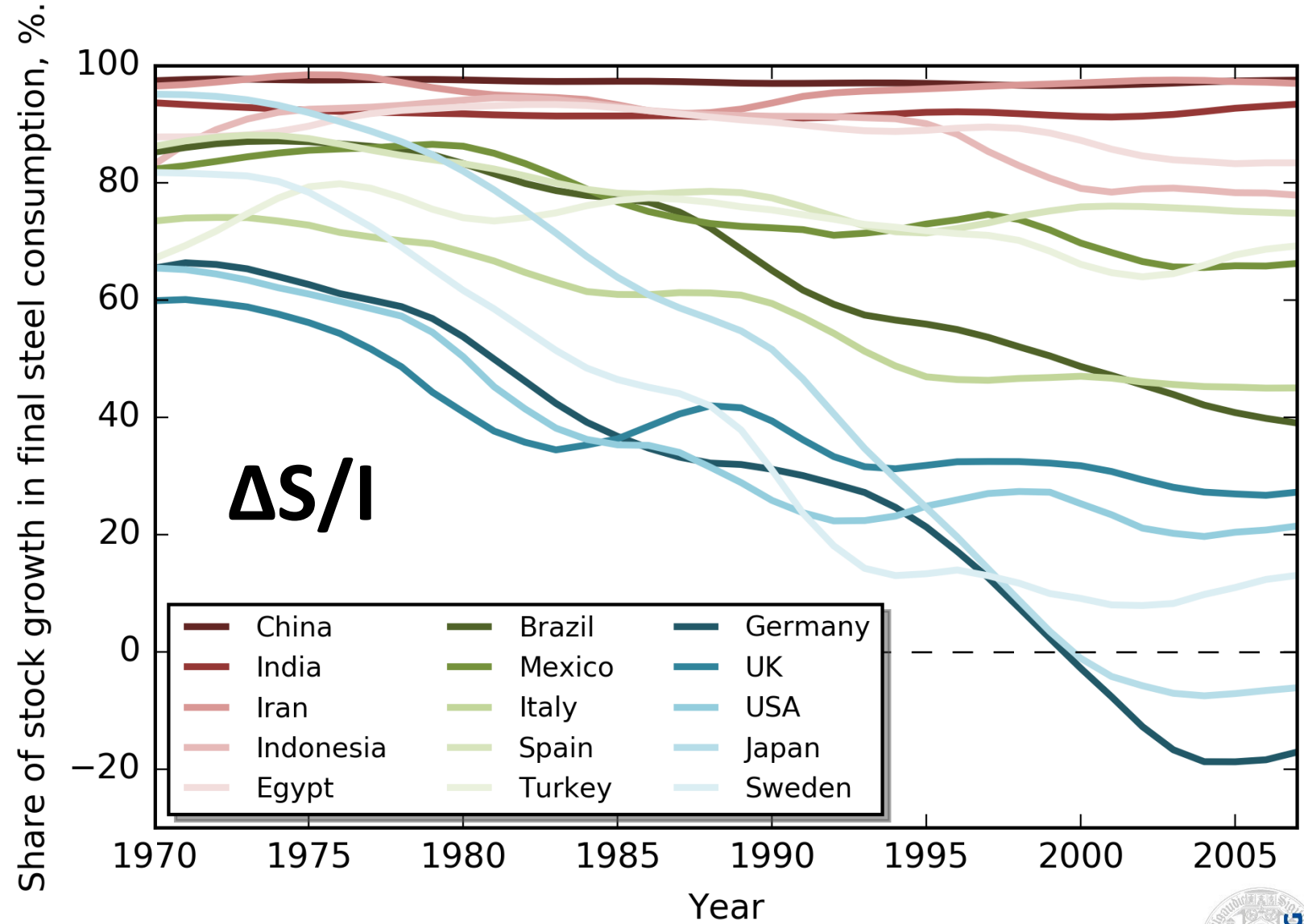
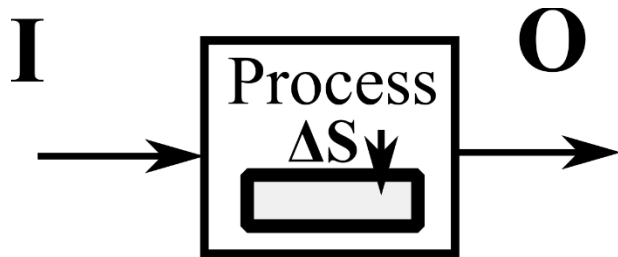
Example: Historic and suggested future per capita stocks based on in-use stock saturation hypothesis



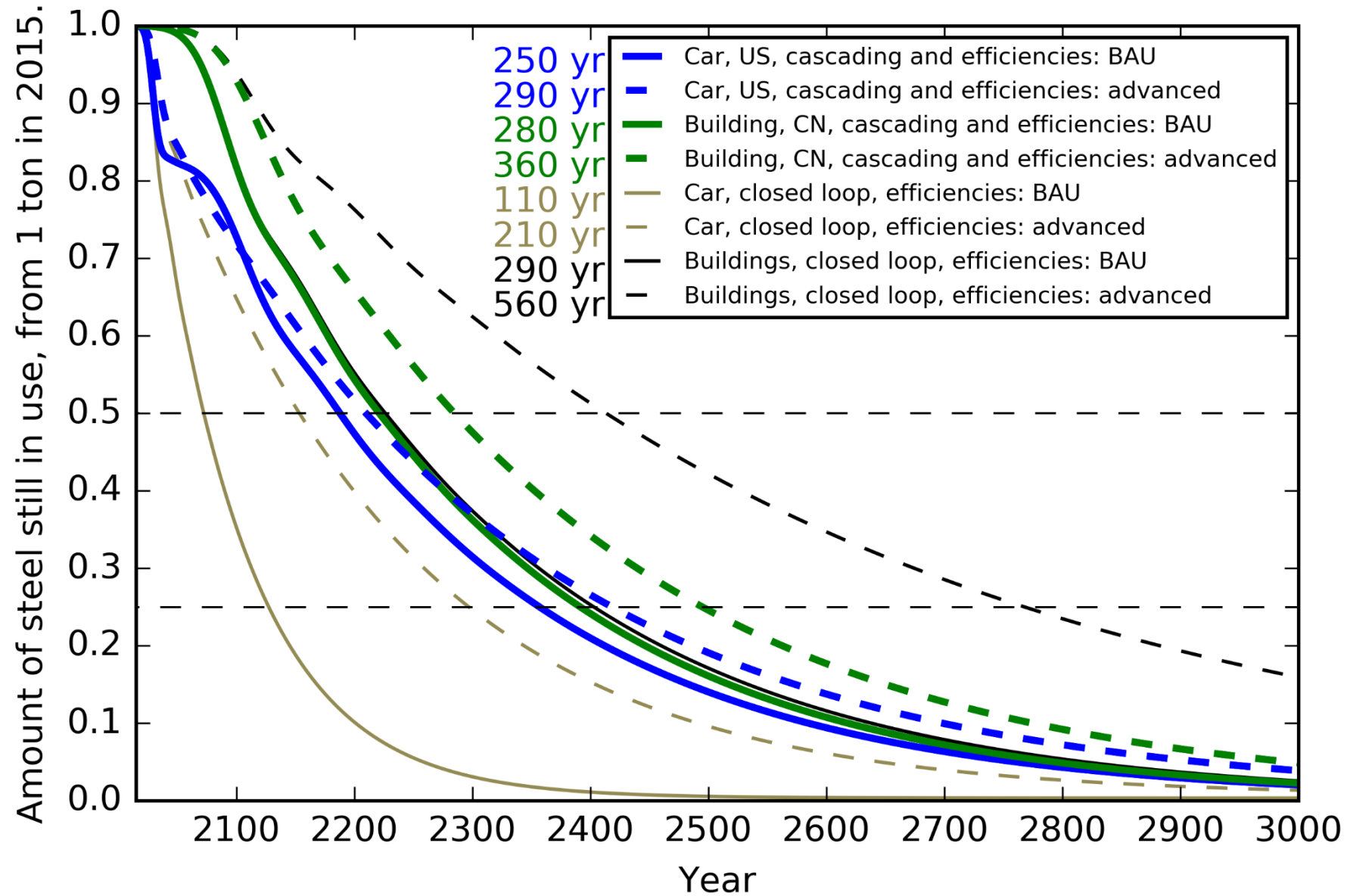
Example Circular economy - Material consumption and stock expansion

From the mass balance of the use phase, one finds that the final material consumption I always has two components: replacement of outflow O and stock expansion ΔS :

$$I = \Delta S + O$$

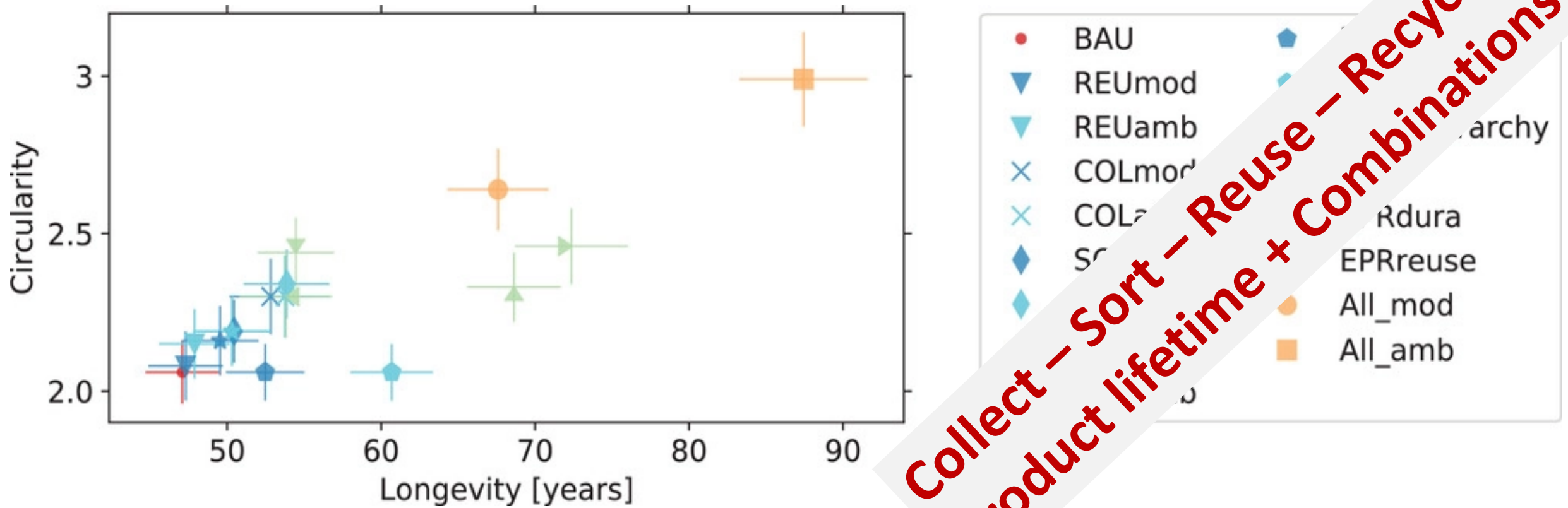


Example: Lifetime of steel in the techno-sphere: 250-300 years



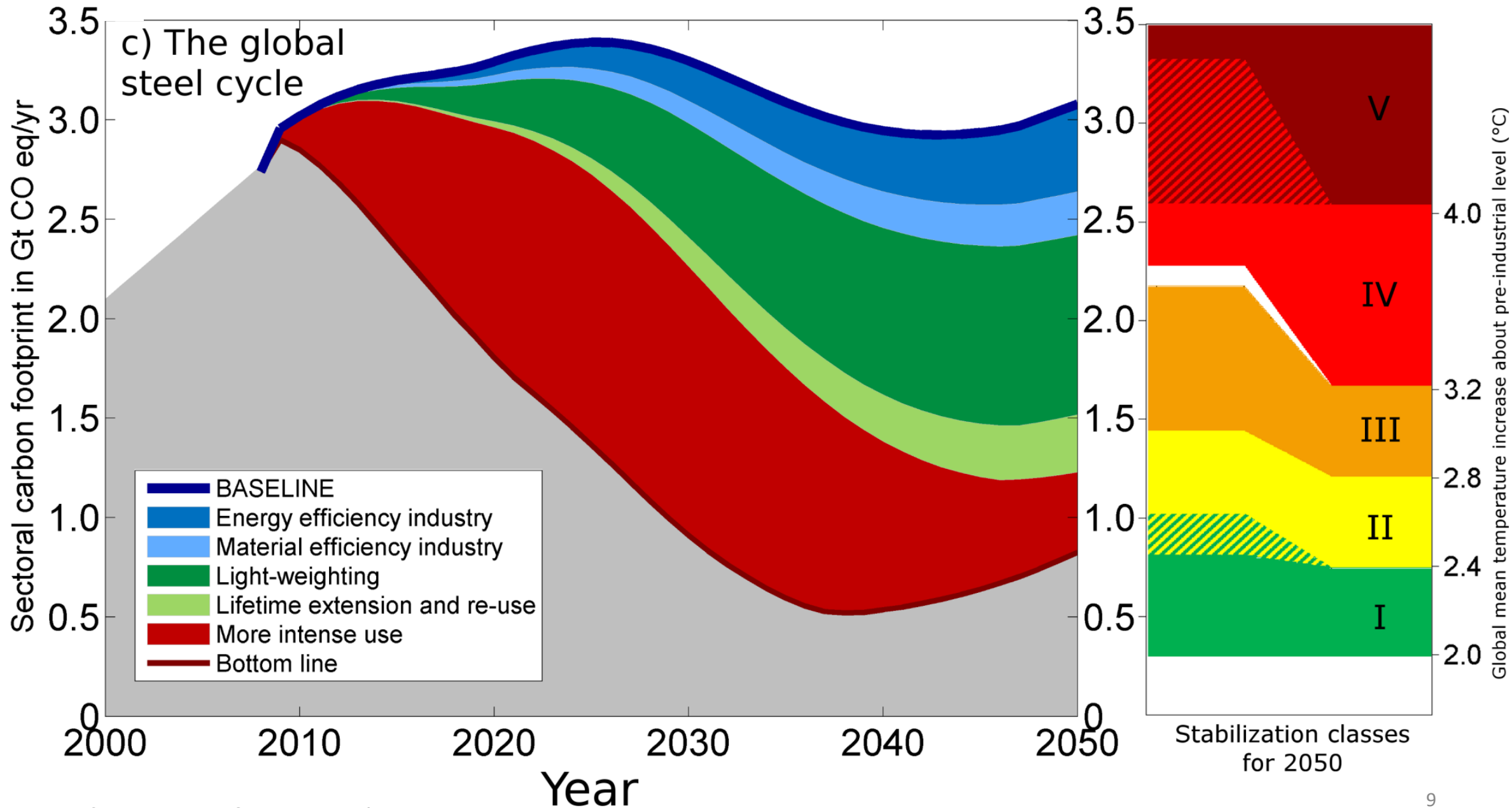
Example - Life cycle perspective: The example of copper

Material efficiency strategies' impact on lifetime in the technosphere (longevity)
And the average number of product life cycles (circularity)



Collect - Sort - Reuse - Recycle
Product lifetime + Combinations

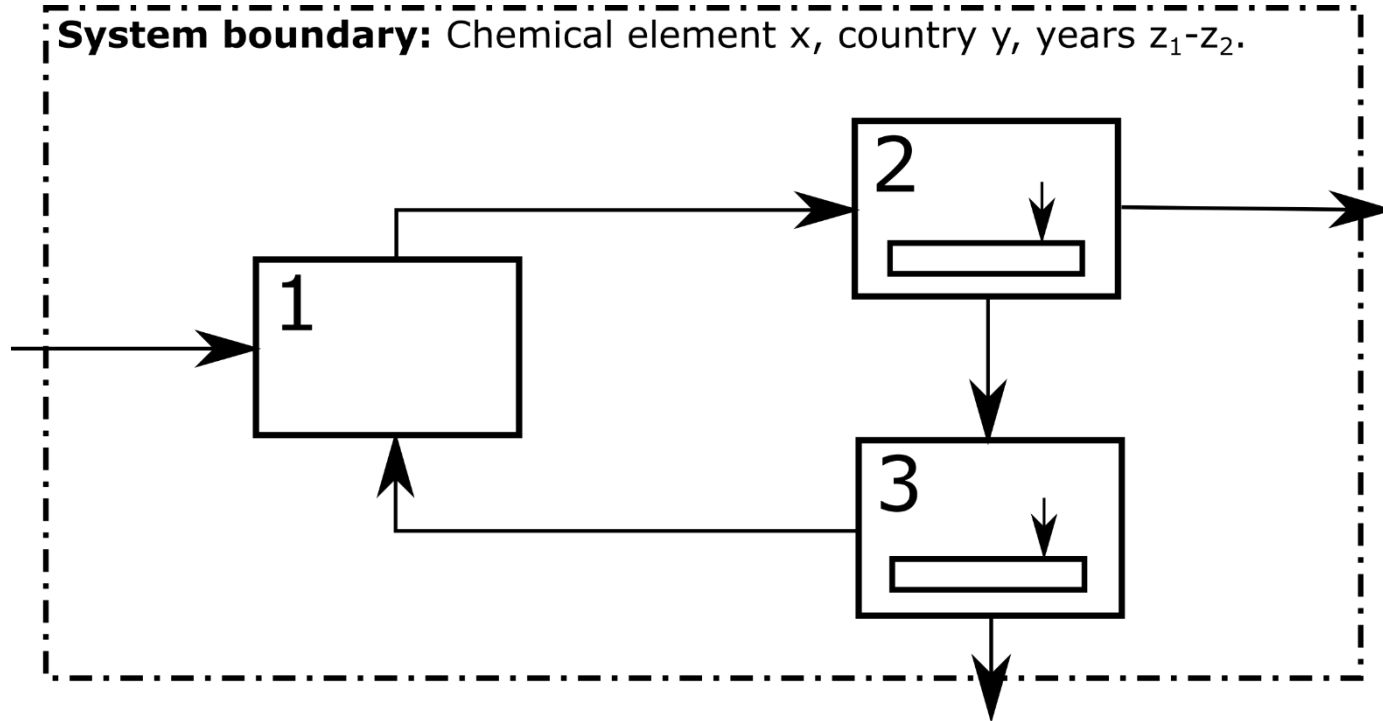
Example: System-wide material and energy efficiency in global steel cycle



Dynamic MFA

Dynamic stock modelling: Basics

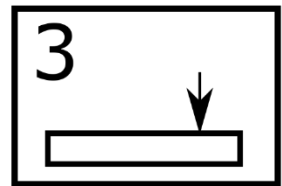
System variables and parameters



System boundary: Chemical element x , country y , years z_1 - z_2 .

 System boundary

 Flow

 Process with ID, stock, and stock change

Stocks, stock changes, and flows together form the **system variables**.

Stocks: $S_1(t)$, $S_3(t)$.

Stock changes (net addition to stock): $\Delta S_1(t)$, $\Delta S_3(t)$.

Flows: $F_{01}(t)$, $F_{12}(t)$, $F_{20}(t)$, $F_{23}(t)$, $F_{31}(t)$, $F_{30}(t)$

A **parameter** is an additional variable that couples different system variables through equations:

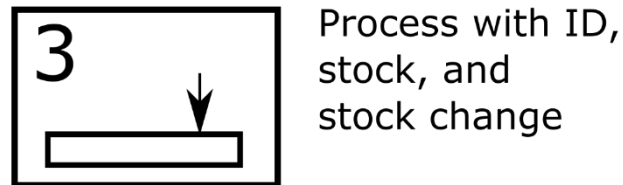
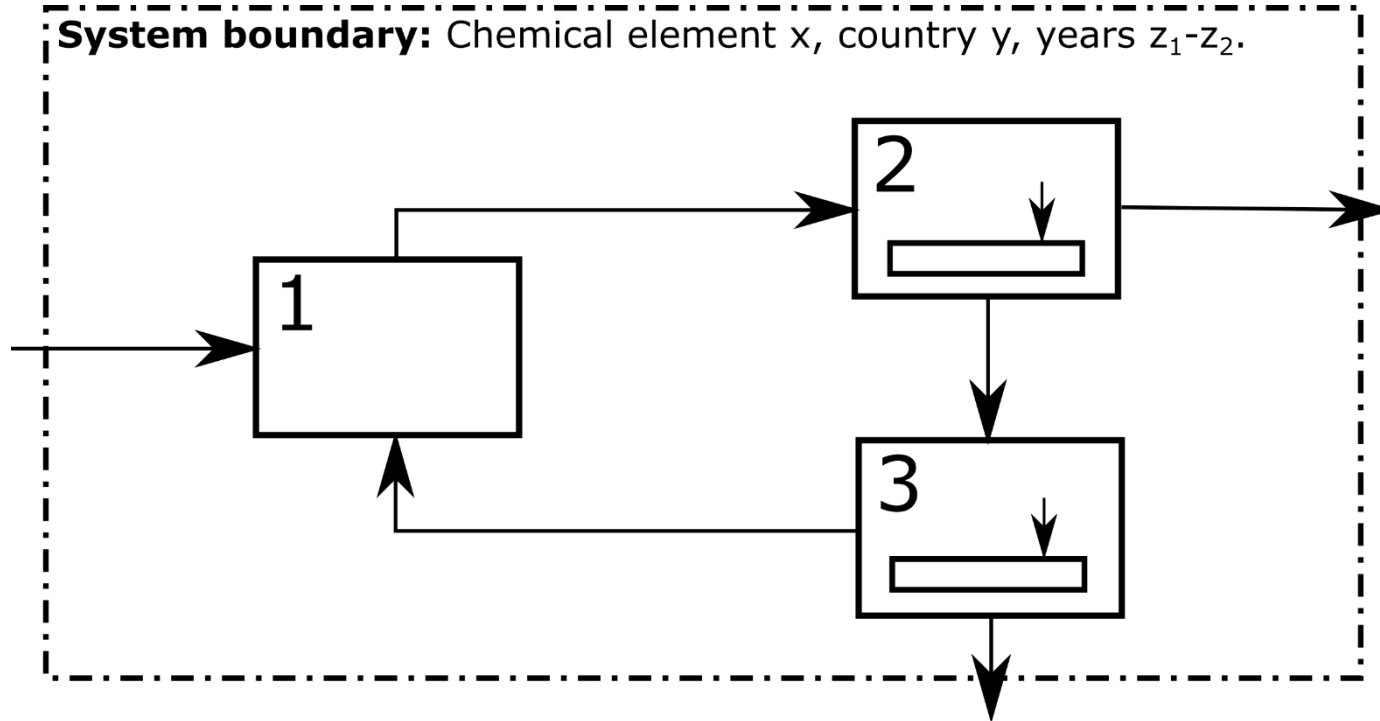
For example:

$$F_{23}(t) = k(t) \cdot F_{12}(t)$$

$$\Delta S_1(t) = (0.15 + 0.01 \cdot t) \cdot F_{12}(t)$$

$$\Delta S_3(t) = 0$$

Process and system balances



For mass, energy, sometimes monetary values,
the process and system-wide balance holds:

Input – Output = Net Stock Change

$$\text{Process 1: } F_{01}(t) + F_{31}(t) - F_{12}(t) = \Delta S_1(t)$$

$$\text{Process 2: } F_{12}(t) - F_{23}(t) - F_{20}(t) = 0$$

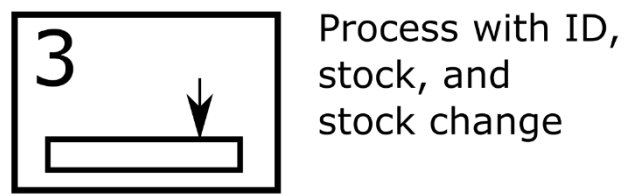
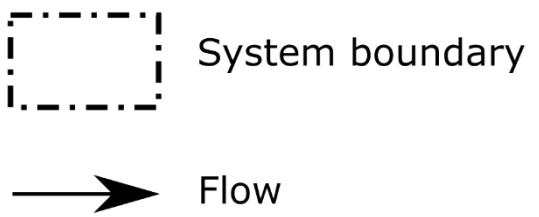
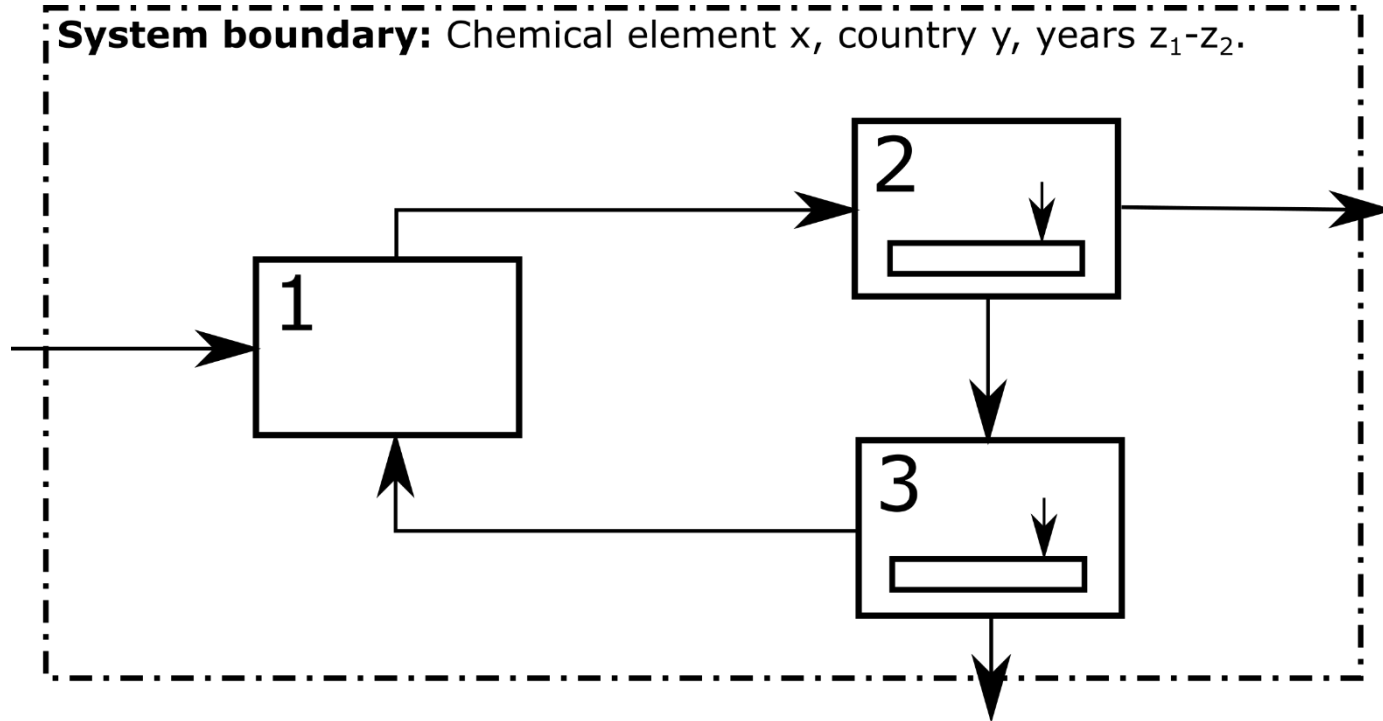
$$\text{Process 3: } F_{23}(t) - F_{31}(t) - F_{30}(t) = \Delta S_3(t)$$

$$\text{System: } F_{01}(t) - F_{20}(t) - F_{30}(t) = \Delta S_1(t) + \Delta S_3(t)$$

For a fully quantified system:

$$\begin{aligned} \# \text{System variables} &= \# \text{balance equations} \\ &+ \# \text{parameters} \\ &+ \# \text{Measurements} \end{aligned}$$

Performance indicators



A major advantage of an explicit system definition is the clear definition of performance indicators.

Efficiency $\eta(t) = \text{useful output}(t) / \text{total input}(t)$

Process 2: $\eta_2(t) = F_{20}(t) / F_{12}(t)$

Process 1: $\eta_1(t) = F_{12}(t) / F_{01}(t)$ OR $\eta_1(t) = F_{12}(t) / (F_{01}(t) + F_{31}(t))$

System: $\eta_S(t) = F_{20}(t) / F_{01}(t)$

Emissions/waste intensity $b = \text{waste} / \text{useful output}$
OR
 $\text{waste} / \text{total input}$

Simple example: Recycled content of steel over time

How has the share of recycled steel in total steel production (proxy for average recycled content of steel in the economy) changed over time?

First, define a system:

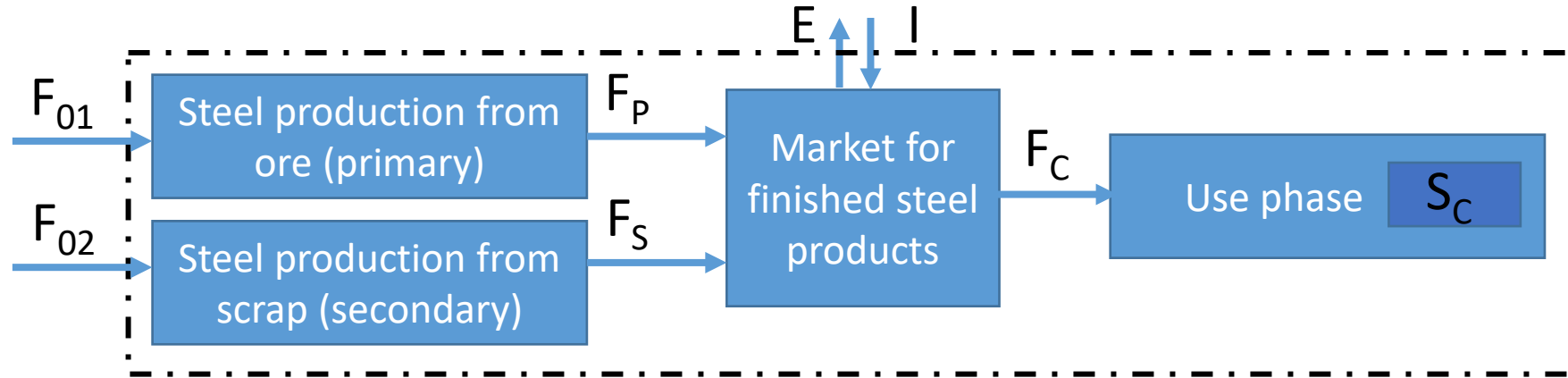
Then, for a quick result, assume that the trade flows all have the same share of primary vs.

secondary steel, so that the average recycled content RC equals the share of secondary in total steel production.

The flows are available from a scenario:*

$$RC(t) = F_S(t) / (F_S(t) + F_P(t))$$

In this simple scenario (cf. table), the recycled content first stays at the current 32% and later increases to 55% in 2025 and 66% in 2050.



Steel production and use, Germany, 2020-2050, reference scenario

Year	F_S , scrap-EAF (Mt/yr)	F_P , BF-BOF (Mt/yr)	RC (%)
2020	14	30	31,8
2025	14	30	31,8
2030	14	30	31,8
2035	14	30	31,8
2040	14	30	31,8
2045	24	20	54,5
2050	29	15	65,9

Snapshot from a quick implementation in a spreadsheet calculator.



*) Data source for scenario: Harpprecht et al. (2022): <https://doi.org/10.1016/j.jclepro.2022.134846>

Dynamic MFA

Types of dynamic models I

General: Population balance model

The stationary model

The leaching model

The fixed lifetime model

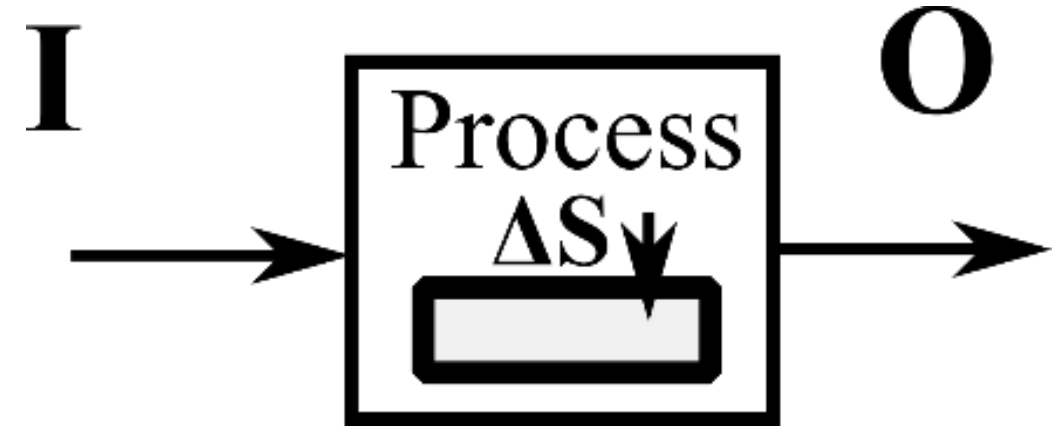
General dynamic stock model $S = S(t)$: Population balance model

The mass balance of a process with inflow $I(t)$, stock $S(t)$, and outflow $O(t)$ leads to the following relation between inflow, outflow, and stock change of a process:

$$\frac{dS(t)}{dt} = I(t) - O(t)$$

$$S(t) = \int_{t_0}^t \frac{dS(\tau)}{d\tau} d\tau$$

The first relation, the population balance model, applies to all processes with a conservation law for a given quantity (energy, material, products, people), the 'population' of that quantity in the process.



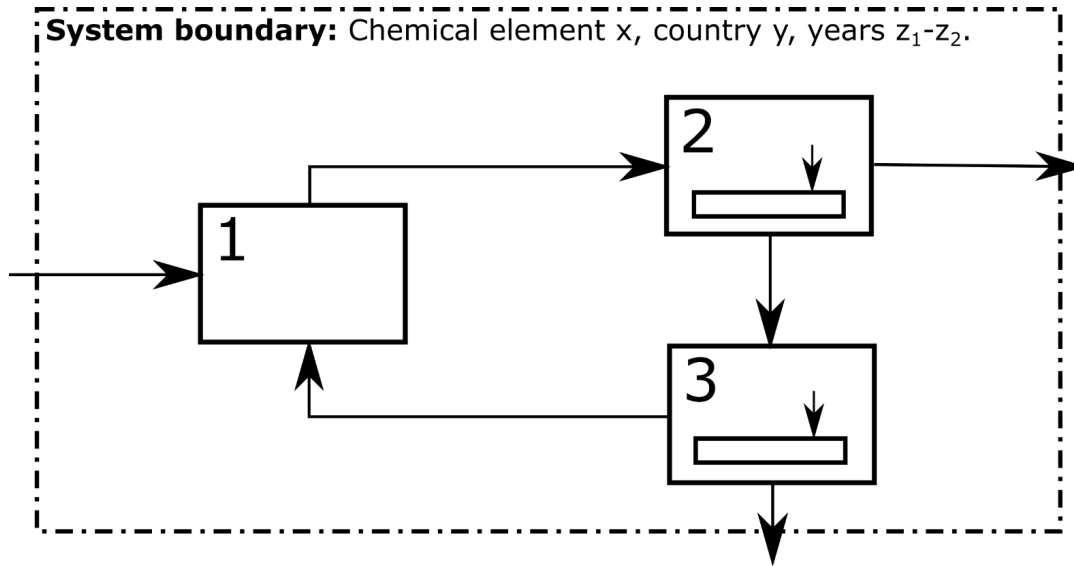
Different reading of the population balance model:

$$I(t) = O(t) + dS/dt$$

All inflow into a process either compensates losses (outflow) or leads to stock growth (dS/dt)

The population balance model (in MFA: mass balance equation) is the basis of all dynamic stock models.

The stationary model: All flows are constant, $F_x(t) = \text{const.}$



The stationary model is the simplest dynamic MFA model:

The Flows of the static MFA are simply extrapolated into the future, leading to constant stock changes and constant increase or decrease of stocks over time:

All flows: $F_{01}(t), F_{12}(t), F_{20}(t), F_{23}(t), F_{31}(t), F_{30}(t) = \text{const.}$

→ All stock changes (net addition to stock): $\Delta S_1(t), \Delta S_3(t) = \text{const.}$

→ All stocks: $S_1(t), S_3(t)$ change constantly over time:

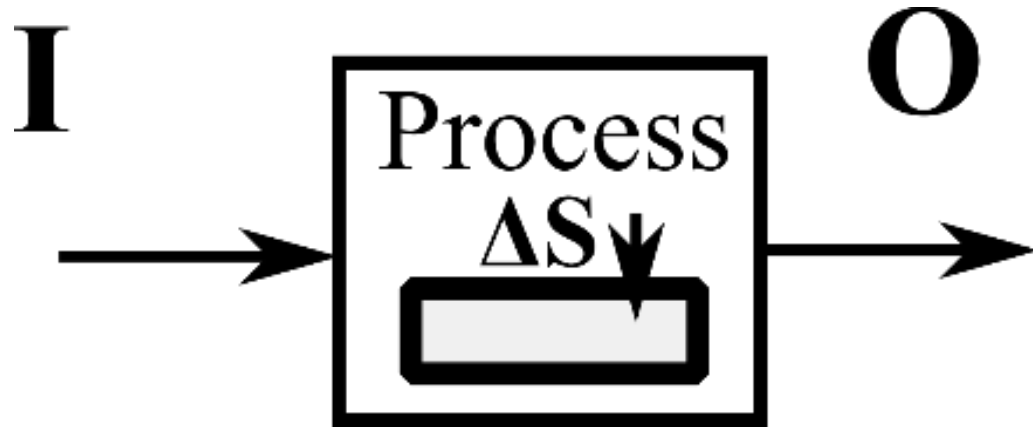
$S_1(t) = S_1(t-1) + \Delta S_1(t)$ etc. (consequence of population balance model in this particular situation)

Applications:

• Simple extrapolations into the future:

- When connecting a battery device to a charger with known capacity and charging rate, the charging time can simply be estimated by a stationary energy flow model: *Capacity = charging time * charging rate*
- The so-called static depletion model for mining. Here, the "Static depletion time" refers to a simple indicator calculated as the ratio of reserves of a mine to the current production level, assuming depletion of the reserve (stock). This simple model estimates how long a known reserve would last at the current rate of extraction and provides a benchmark for assessing resource availability.
- The range estimator of a gasoline car calculates the remaining km that can be driven with the fuel left in the tank and the momentary rate of fuel combustion.

The leaching model for a stock:



Model equation:

$$O(t) = c \cdot S(t)$$

At any given time, the outflow is proportional to the total stock in the process. The outflow *leaches from the stock*, with a leaching rate or decay rate or decomposition rate c . c is a percentage per time, e.g. 5%/yr.

Simplification:

If no further inflow ($I(t) = 0$), the initial stock S_0 decays exponentially: $S(t) = S_0 \cdot \exp(-c \cdot t)$

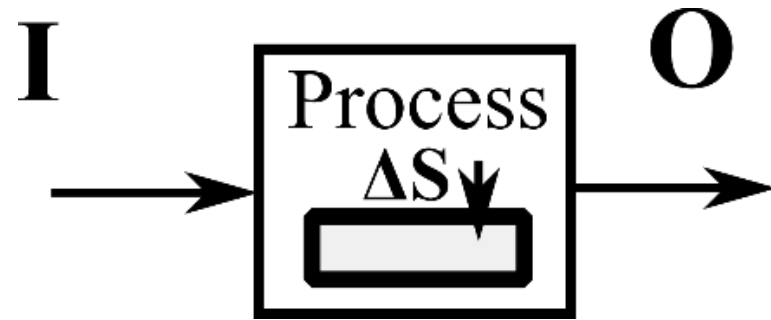
Examples:

- Radioactive decay: rate of atoms decaying is proportional to number of atoms present
- Decomposition of alcohol or other drugs in human body
- Partial absorption of CO_2 in atmosphere by forests and oceans leads to several combined exponential decay functions
- Leaching from or decomposition of deposits of pollutants or nutrients in landfills (methane), tailings (heavy metals), or soil (nitrogen).

Applications:

- Easy to apply: only 2 parameters needed: S and c
- Only works for processes with no 'internal memory': probability of an item leaving the stock is independent of its residence time.

The fixed lifetime model for a stock



Model equation:

$$O(t) = I(t - T)$$

At any given time t , the outflow equals the inflow of $t-T$. The outflow simply is the inflow with a delay of the lifetime T .

The stock is then (mass balance) simply the sum of all inflows between now (t) and $t-T+1$:

$$S(t) = \sum_{\tau=t-T+1}^{\tau=t} I(\tau)$$

Applications:

- Simple way of estimating how a stock accumulates as a result of inflows from previous years: E.g., estimate the number of cars/laptops/PV panels in a country based on sales statistics and a lifetime estimate.
- Simple way of estimating how much scrap and recycling will be possible in the future, based on recent and current sales: The future number of end-of-life vehicles/laptops/PV panels that will be available for recycling in year t roughly equals the sales volume of these devices in year $t-T$, where T is the average lifetime.

Dynamic MFA

Excursus:

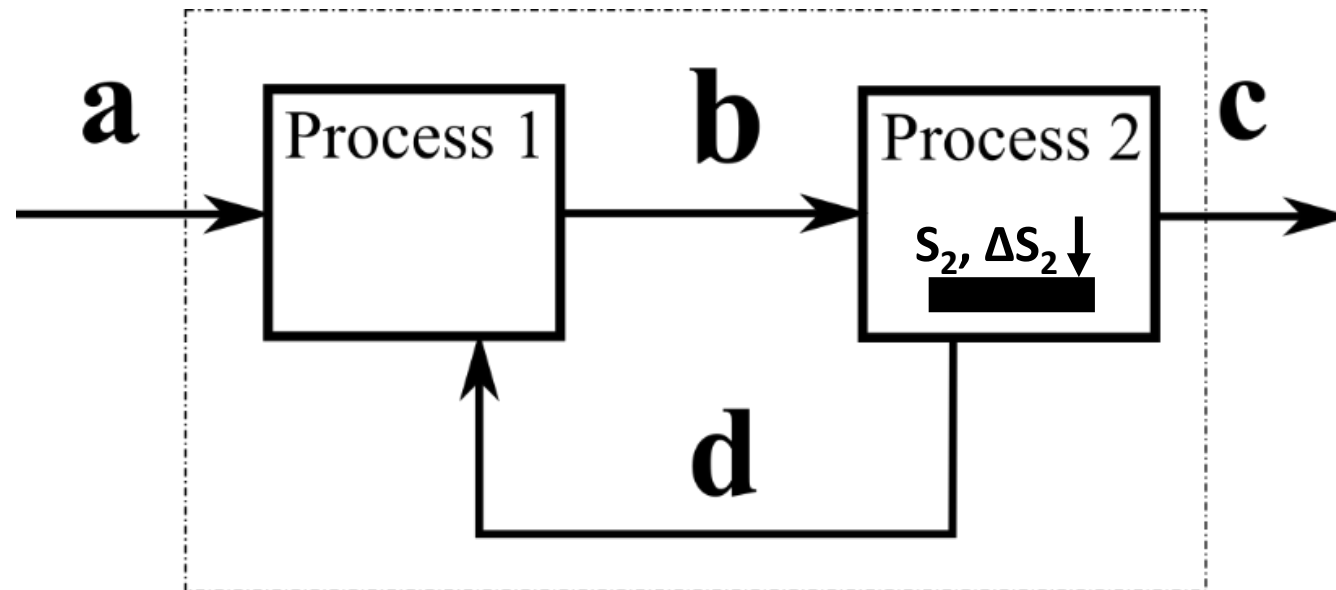
Solving mathematical dMFA models, examples

Solving dynamic MFA systems: (Linear) difference equations

Idea: Level of system variable x at time t is determined by previous states:

$$x(t) = a_1 \cdot x(t-1) + a_2 \cdot x(t-2) + \dots + a_n \cdot x(t-n)$$

Example:



Growth in demand:

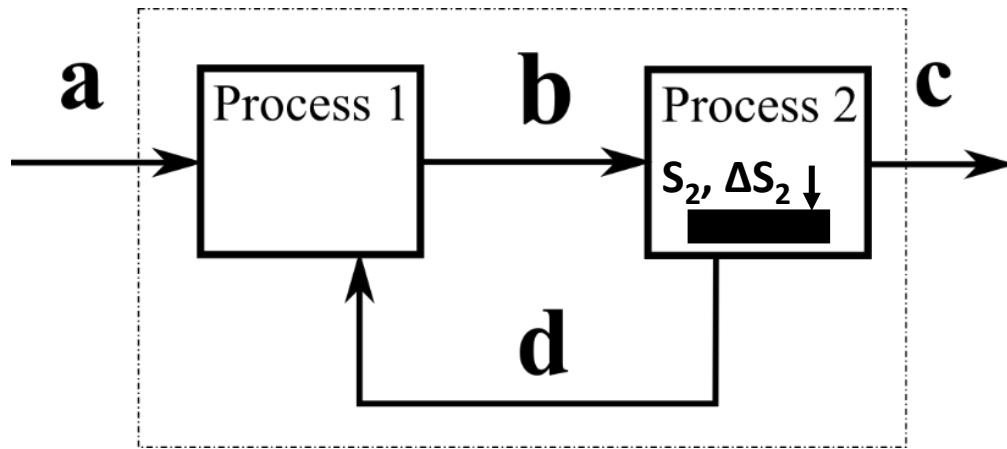
$$b(t) = (1+\alpha) \cdot b(t-1)$$

Scrap flow, fixed lifetime T in S_2 :

$$d(t) = \beta \cdot b(t-T)$$

Solving dynamic MFA systems: (Linear) difference equations

To solve the example we need a complete set of starting values for $t = 0$.



Starting values: $b(0) = B, b(t < 0) = 0$

Model solution:

$$b(t) = (1+\alpha)^t \cdot B$$

$$d(t) = 0 \text{ (f. } t < T) \text{ and } \beta \cdot (1+\alpha)^{t-T} \cdot B \text{ else}$$

$$a(t) = (1+\alpha)^t \cdot B \text{ (f. } t < T) \text{ and} \\ (1+\alpha)^t \cdot B - \beta \cdot (1+\alpha)^{t-T} \cdot B \text{ else}$$

$$c(t) = 0 \text{ (f. } t < T) \text{ and } (1-\beta) \cdot (1+\alpha)^{t-T} \cdot B \text{ else}$$

$$S_2(t) = ((1+\alpha)^t + (1+\alpha)^{t-1} + \dots + (1+\alpha)^{t-T}) \cdot B \\ \text{(for } t \geq T)$$

- System equations:**
- Growth in demand:

$$b(t) = (1+\alpha) \cdot b(t-1)$$
 - Scrap flow, fixed lifetime T in S_2 :

$$d(t) = \beta \cdot b(t-T)$$
 - Mass balances:

$$a(t) + d(t) = b(t)$$

$$b(t) = c(t) + d(t)$$

Solving dynamic MFA systems: Differential equations

In many practical cases the system equations of a dynamic MFA model can be formulated as differential equations, which are equations that have the derivatives of the stocks and flows as variables.

Basic example: Exponential decay or growth of a system variable X:

$$\frac{dX}{dt} = \gamma \cdot X \quad , \quad \text{of which a solution is} \quad X(t) = X_0 \cdot e^{\gamma \cdot t}$$

Application (1) for the decay of an initial stock S_0 according to the leaching model:

For $\gamma < 0$: Exponential decline, e.g.,
slowly decaying stock ('leaching model')

$$S(t) = S_0 \cdot e^{-|\gamma| \cdot t}$$

Application (2) for exponential growth of an inflow:

For $\gamma > 0$: Exponential growth, e.g.,
Exponentially growing consumption

$$I(t) = I_0 \cdot e^{\gamma \cdot t}$$

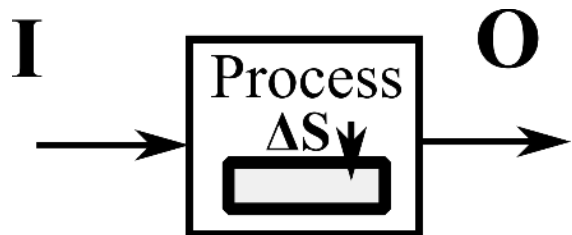
Solving dynamic MFA systems: Logistic growth

To curb the exponential growth, the growth term dS/dt can be limited as follows:
 With increasing stock S , the factor $1-S$ goes to 0 as S approaches 1 , curbing growth.

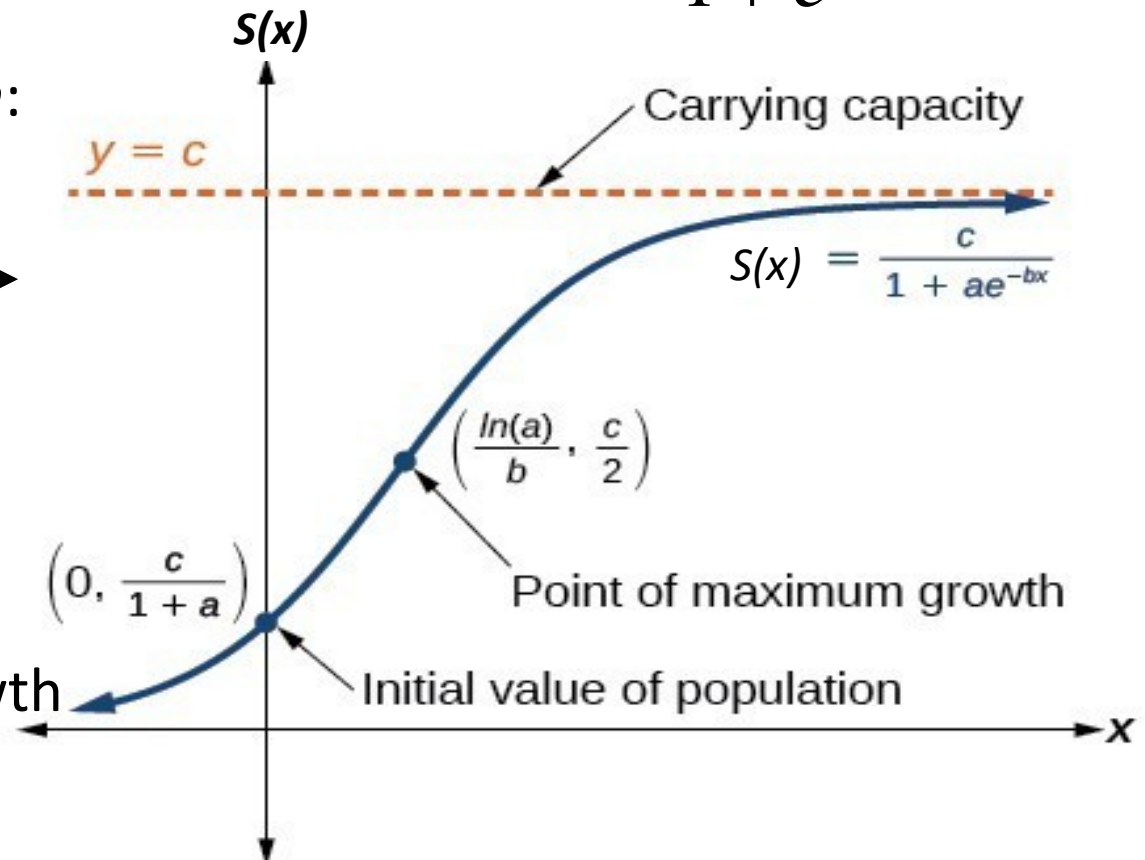
$$\frac{dS}{dt} = \gamma \cdot S \cdot (1 - S) , \quad \text{of which a solution is} \quad S = \frac{1}{1 + e^{-t}}$$

More general, the *logistic growth equation*:

$$\frac{dS}{dt} = b \cdot S \cdot \left(1 - \frac{S}{c}\right) \longrightarrow$$



Stock growth exponentially first, then growth slows down and stock saturates.



Dynamic stock modelling

Building realistic models by tracing age-cohorts and including lifetime distributions

Introduction: Age-cohorts of products in the use phase

The energy efficiency and material composition of products changes over time.

→ *When linking products in the use phase to energy and materials, we need to know the breakdown into different technologies.*

This information is commonly obtained by tracing the *production year (aka age-cohort or vintage)* of the products/vehicles in the use phase.

→ At any given point in time, we not only know the total stock but also its breakdown into different production years with their respective energy efficiency and material composition.

→ The same applies to the outflow: Dividing the total outflow / end-of-life goods into age-cohorts allows to accurately estimate the amount of different materials present.

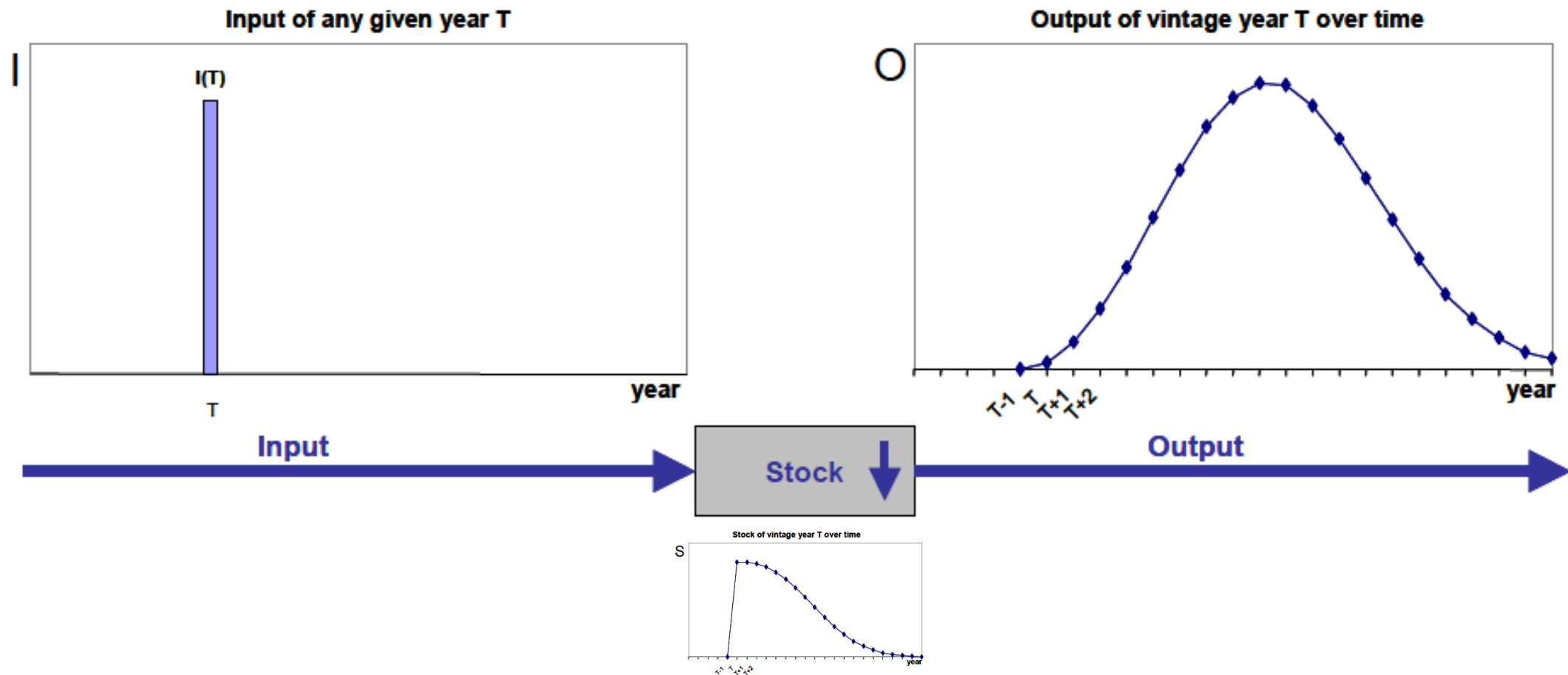
The breakdown of stocks into age-cohorts enables realistic and accurate modelling of the stock-flow-service nexus. It requires more data and makes calculations more complicated.



Introduction: Distributed lifetime model for an input pulse: Single age-cohort

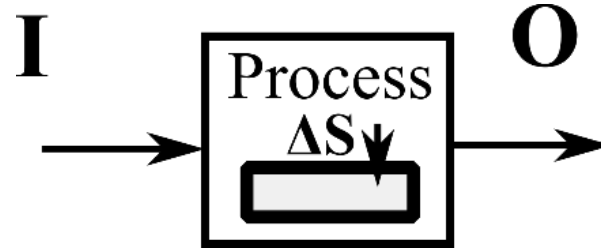
Most products don't have a fixed lifetime but show some early failures, typical lifetime, and some very long-lasting exceptions.

→ We consider this diversity in the lifetime of individual products by modelling a lifetime distribution rather than a fixed lifetime:

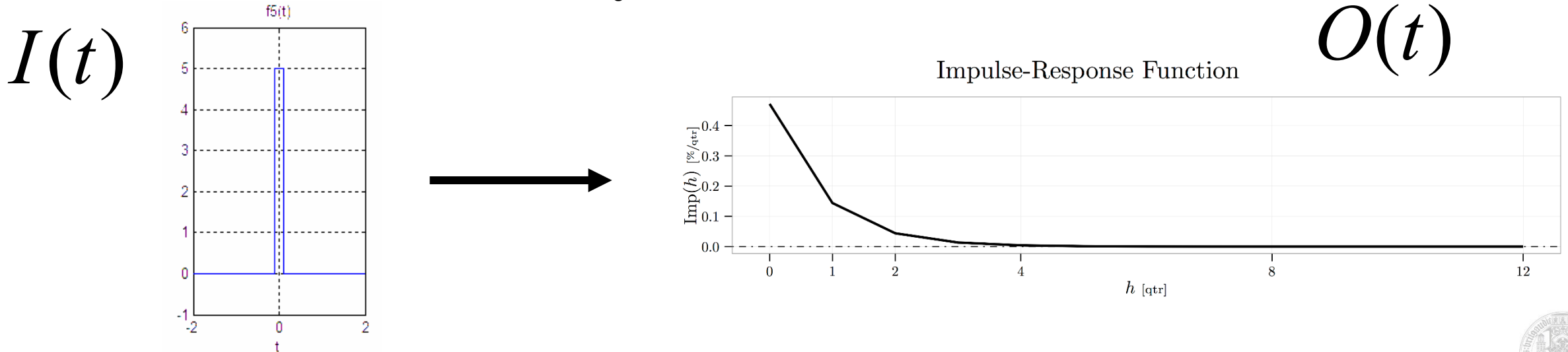


General lifetime modelling: The impulse response function of a process

The **impulse response**, or **impulse response function (IRF)**, of a dynamic system is its output when presented with a brief input signal, called an impulse.

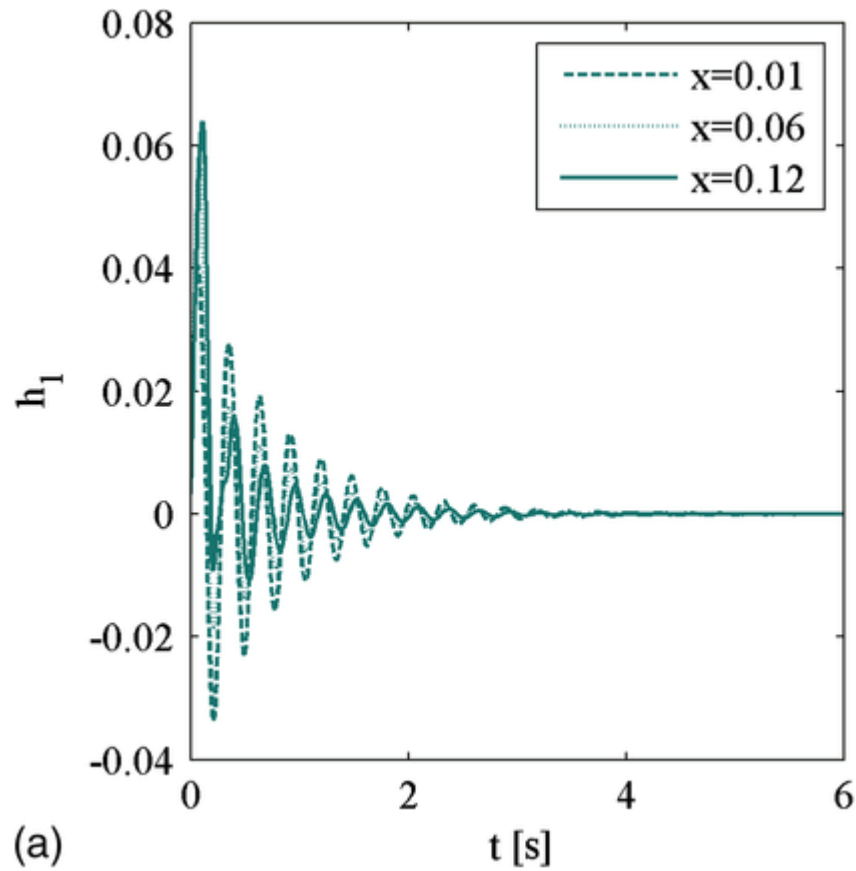


The impulse response function of a process with a stock is the outflow $O(t)$ as response to an instantaneous inflow $I(0) = I_0$ at time $t = 0$.

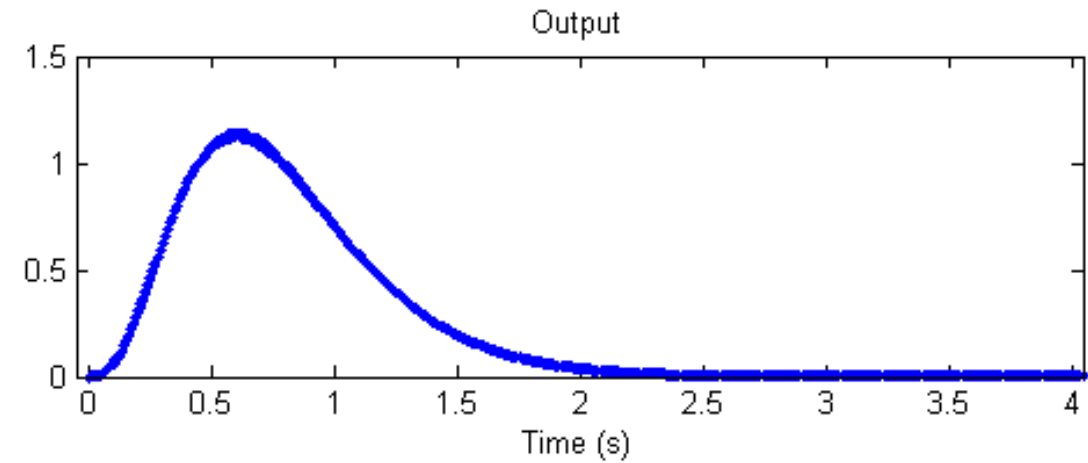


Types of impulse responses

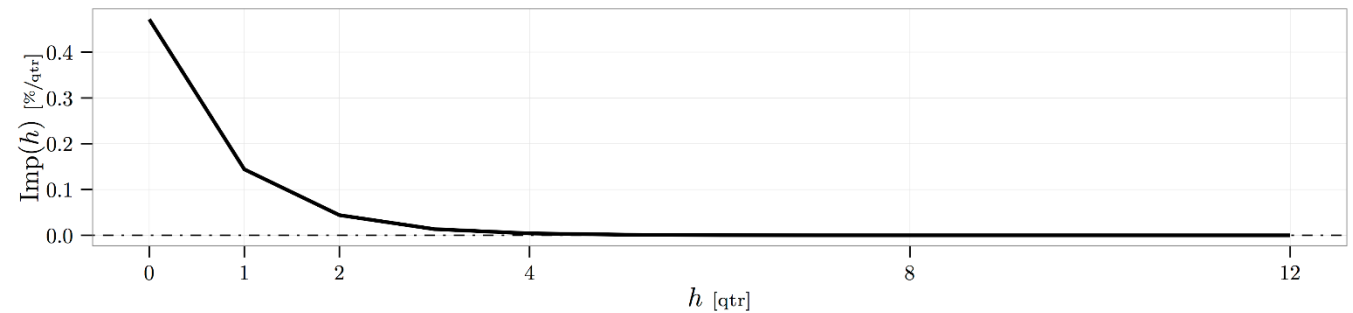
Oscillating



Delayed response



Decaying response



Response to an inflow: Age-cohorts and lifetime

Two types of response are commonly considered in dynamic stock modelling:

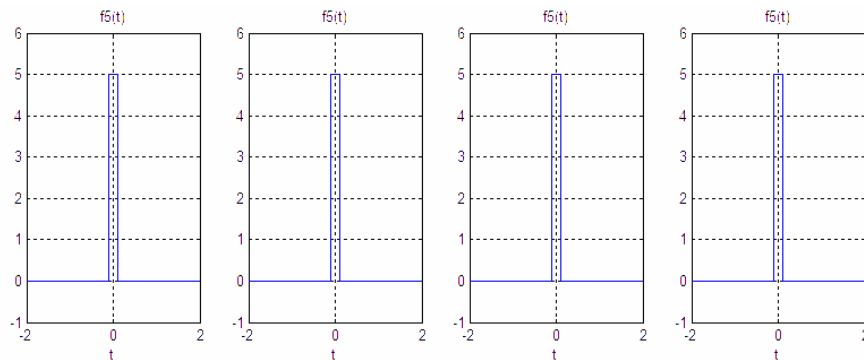
- Delayed response → lifetime model
- Decay → leaching model

The response of the stock is linear: The responses to different inflows (input pulses) are simply added up.

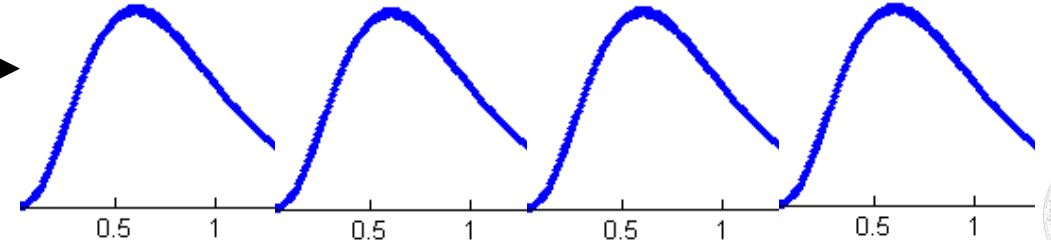
(analogy: The sound of an orchestra is the superposition ('sum') of the sounds of the different musical instruments.

Counter-example: distortions added to guitar sounds in rock music)

$I(t)$



$O(t)$



Tracking inflows/additions to stock: Age-cohorts and lifetime

For linear dynamic stock models (standard situation):

- Each input to stock (inflow of a given year) can be traced separately, and the fraction of the stock that originates from a given input at time t is called the *age-cohort* (of) t .
- The different age-cohorts can be traced separately: At any time, the total stock can be broken down into materials/products from different age-cohorts.

$$S(t) \rightarrow S(t, c)$$

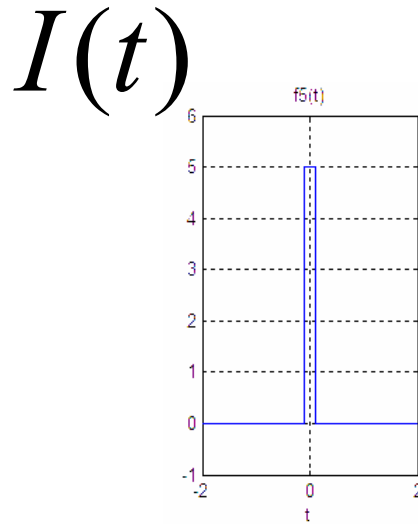
- The time series of the stock (column) on the right side is the row sum of the *stock table*, which breaks down the total stock $S(t)$ of each year t (row) into different age-cohorts (column).
- Tracing a single age-cohort (column) over time shows its gradual decline.

Year t	Age-cohort, starting in the past								Stock (t) (cars)
	-2	-1	0	1	2	3	4	5	
0	4	8	3	0	0	0	0	0	15
1	4	8	3	1	0	0	0	0	16
2	3	8	3	1	5	0	0	0	20
3	2	6	2	1	5	6	0	0	22
4	1	3	2	0	5	6	3	0	20
5	0	1	1	0	4	5	3	7	21

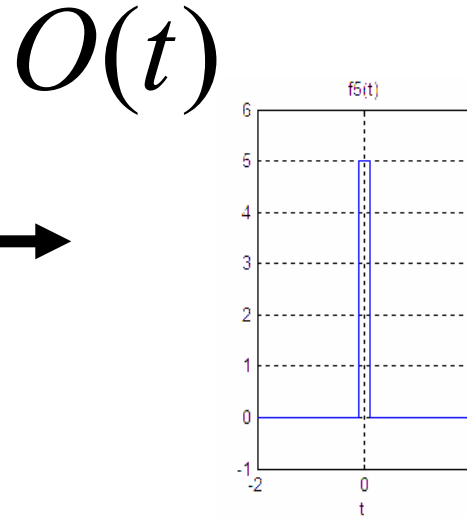
Fixed and distributed lifetimes

For a delayed response to an inflow (e.g. duration of product use) two cases are distinguished:

**Fixed
Product
lifetime**



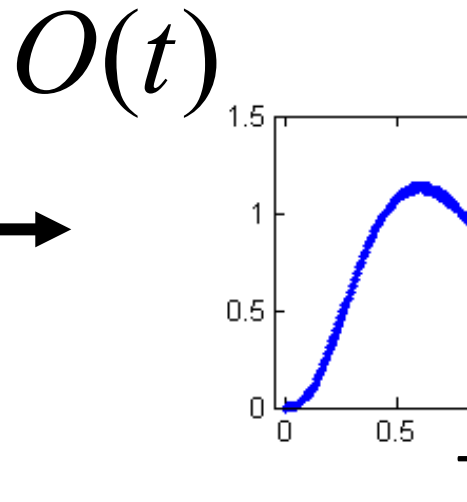
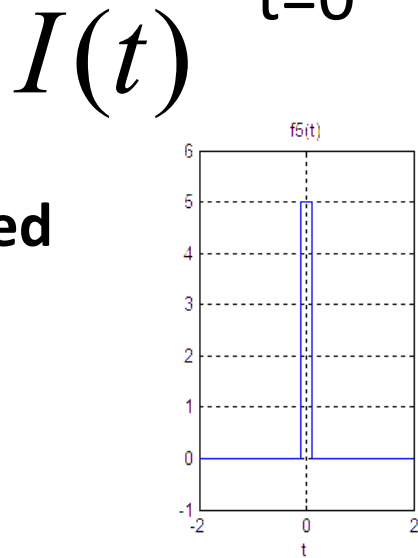
$t=0$



$t=T$

For fixed product lifetimes the entire age-cohort leaves the stock together after the lifetime T has expired. For the distributed lifetime the different fractions of the age-cohorts leave the stock with different Probabilities, so that the average lifetime (expectation value) equals T .

**Distributed
Product
lifetime**



The probability distribution (discrete case)

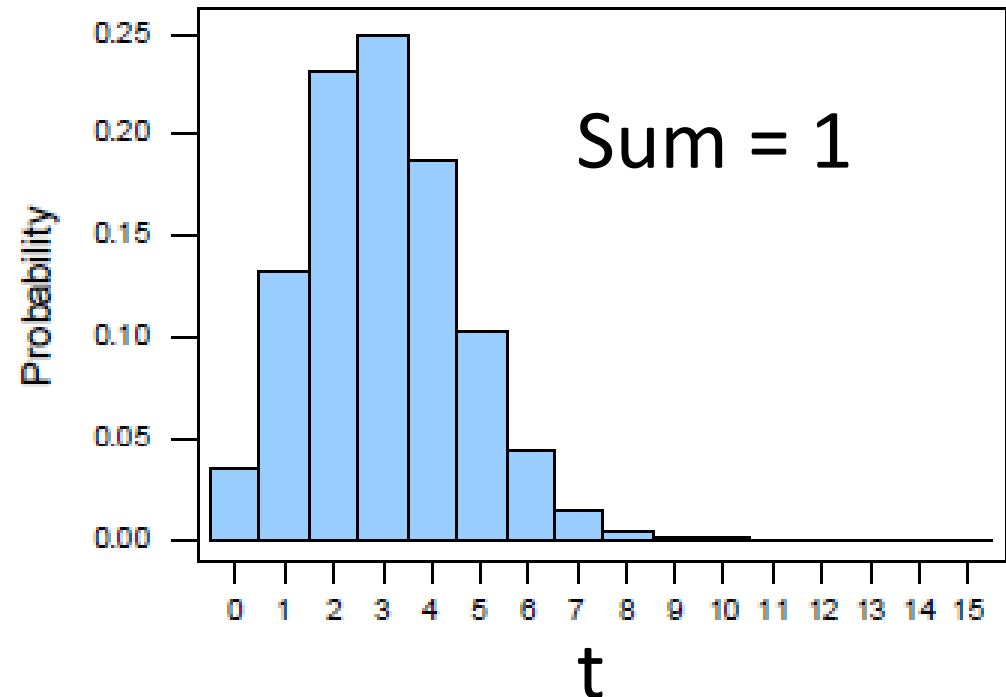
Discrete probability distributions (pd) represent the probabilities for the different discrete events.

In dynamic stock modelling the events under consideration are that the product leaves the stock after time t , and the pd indicates probability of a product becoming obsolete at time t .

For very large stocks (millions of cars etc.) the probability distribution indicates the fraction of the stock leaving.

Unit: 1

Binomial distribution with $n = 15$ and $p = 0.2$

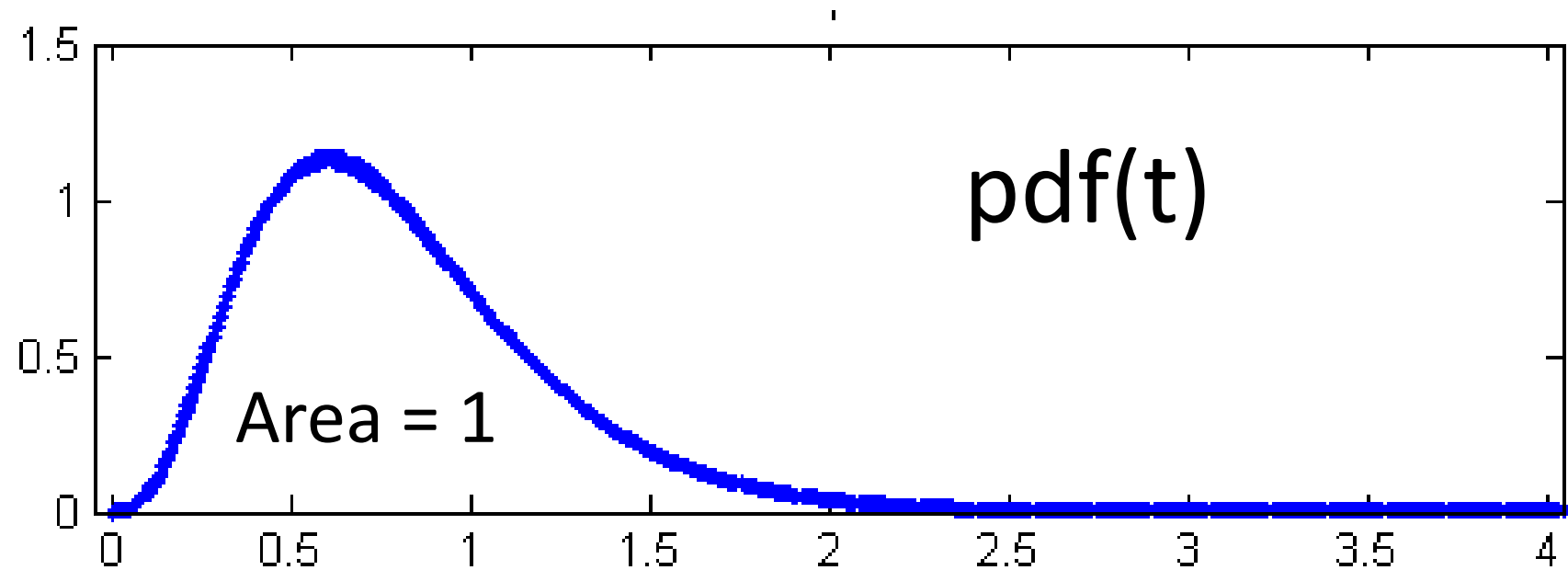


The probability density function (pdf)

Continuously distributed lifetimes are the norm in dynamic stock modelling, as they reflect reality best (age-cohorts of vehicles, buildings, electronic devices...)

If we take the outflow of an inflow pulse with a continuously distributed lifetime and normalize the curve so that the area is 1, we arrive at the *probability density function (pdf)* of the lifetime model.

Unit = 1/time
→ pdf(t)
indicates rate
of decay



The lifetime model for an input pulse: Single age-cohort

With the pf and the pdf we can recalculate the outflow of an input pulse as

$$O(t) = I_0 \cdot pd(t - t_0) \quad (\text{discrete case, } I_0 \text{ is measured as a rate in kt/yr})$$
$$[kt / yr] = [kt / yr] \cdot 1$$

$$O(t) = I_0 \cdot pdf(t) \quad (\text{continuous case, } I_0 \text{ is measured as amount in kt})$$
$$[kt / yr] = [kt] \cdot [1 / yr]$$

The lifetime model for an input pulse: Multiple age-cohorts

In the case of a time series of input flows, we superpose (add) the outflows from the different age-cohorts:

$$O(t) = \sum_{t_0}^t I(\tau) \cdot pf(t - \tau) \quad (\text{discrete case, } I(t) \text{ is measured as a rate in kt/yr})$$

$$[\text{kt/ yr}] = [\text{kt/ yr}] \cdot 1$$

$$O(t) = \int_{t_0}^t I(\tau) \cdot pdf(t - \tau) d\tau \quad (\text{continuous case, } I_0 \text{ is measured as amount in kt})$$

$$[kt / yr] = [kt / yr] \cdot [1]$$

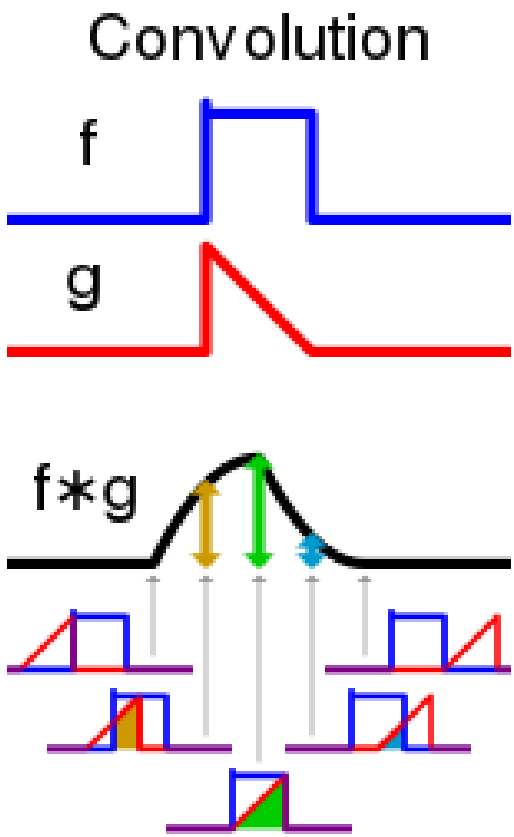
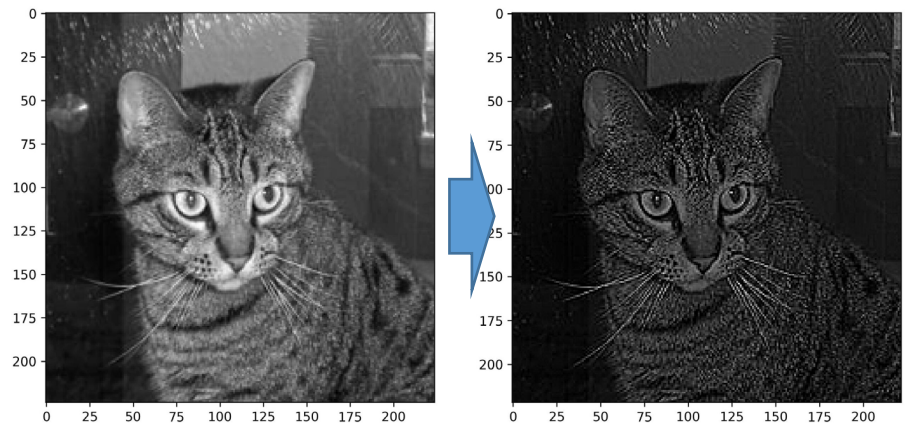
Convolution, the general math operation behind the lifetime model

The mathematical operation used here is called *convolution*, which is the formal notion of applying a filter (here: delay from product lifetime) to an incoming signal.

$$O = I * pdf$$

$$O(t) = \int_{t_0}^t I(\tau) \cdot pdf(t - \tau) d\tau$$

Other example: Applying a filter in sound or image processing:



<https://en.wikipedia.org/wiki/Convolution>

<https://towardsdatascience.com/tensorflow-for-computer-vision-how-to-implement-convolutions-from-scratch-in-python-609158c24f82/>

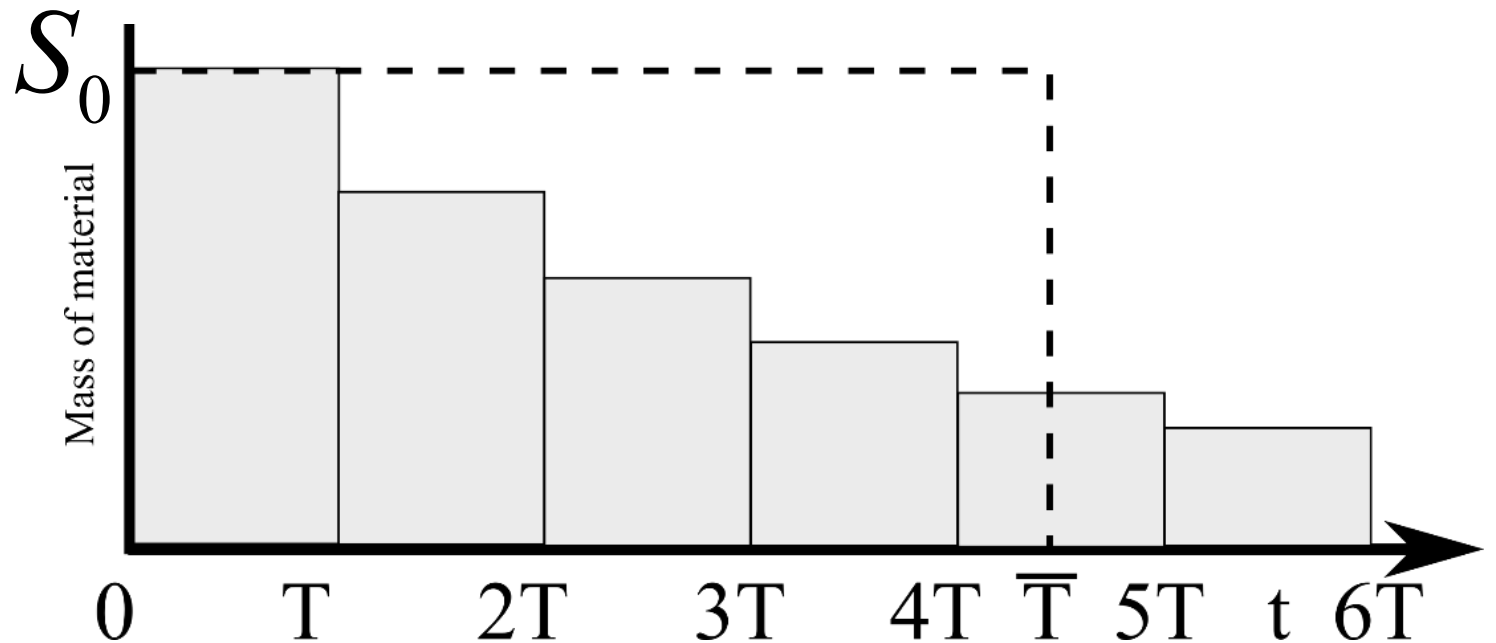


Calculating the average lifetime of an age-cohort

The average lifetime \bar{T} of an age-cohort is defined as the average residence time of the age-cohort in the stock.

Plot: Stock (t) after initial inflow at t=0.

$$\bar{T} = \frac{1}{S_0} \cdot \int_0^{\infty} S(t) dt$$



Read plot vertically: for each time t the stock at t is indicated.

Read plot horizontally: for the different fractions of the stock the total lifetime is indicated, sorted from shortest (top) to longest (bottom).

From combining both perspectives the average lifetime can be derived.

Dynamic MFA

Types of dynamic models II

The inflow-driven model:

Introduction and software exercise

The inflow-driven model: Research questions

In-use stocks of buildings, vehicles, and infrastructure provide services to people and are a central determinant of sustainable development.

For many stocks, in particular, material stocks estimates are not available.

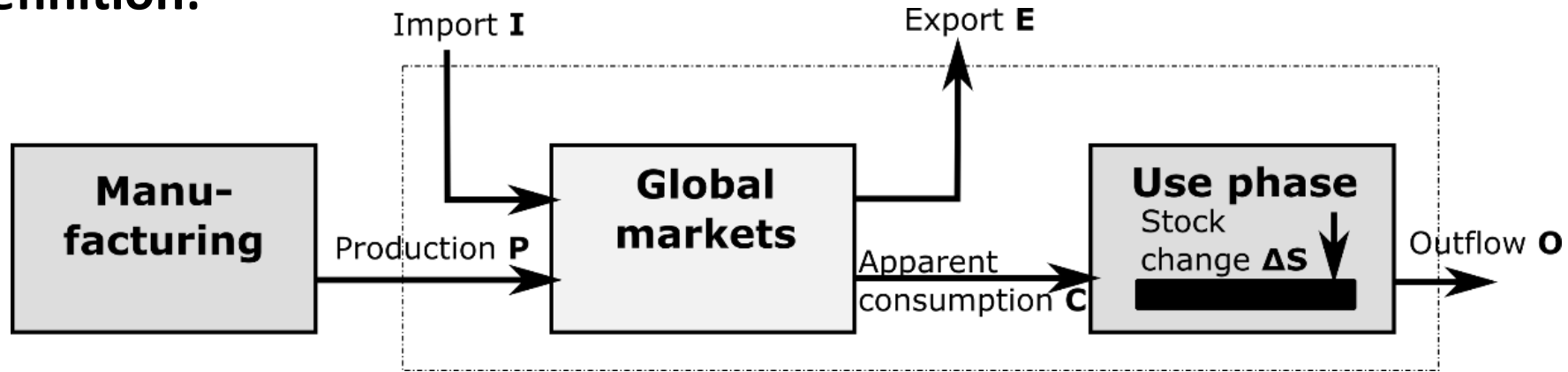
Material stocks can be determined

- a) 'bottom-up': from building and vehicle statistics and product-specific material content data
- b) 'top-down': from aggregated consumption data and a lifetime model



Applying the inflow-driven model to estimate in-use stocks

System definition:



1) Determine apparent consumption:

$$C = P + I - E$$

2) Apply the convolution:

$$O(t) = \int_{t_0}^t C(\tau) \cdot pdf(t - \tau) d\tau$$

3) Determine stock change

$$\Delta S(t) = C(t) - O(t)$$

4) Determine stock

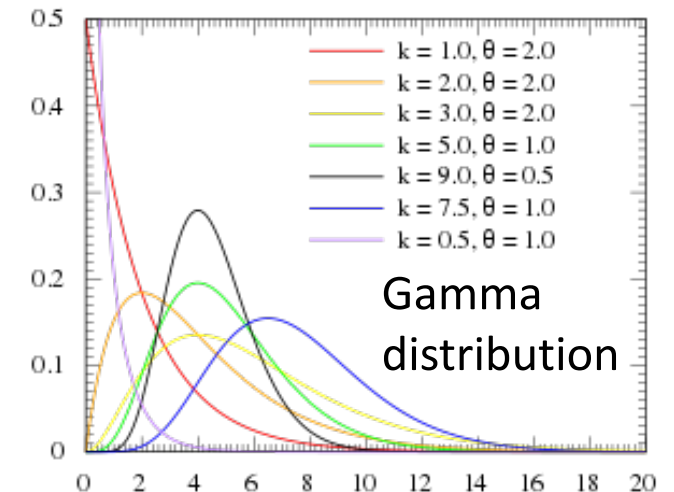
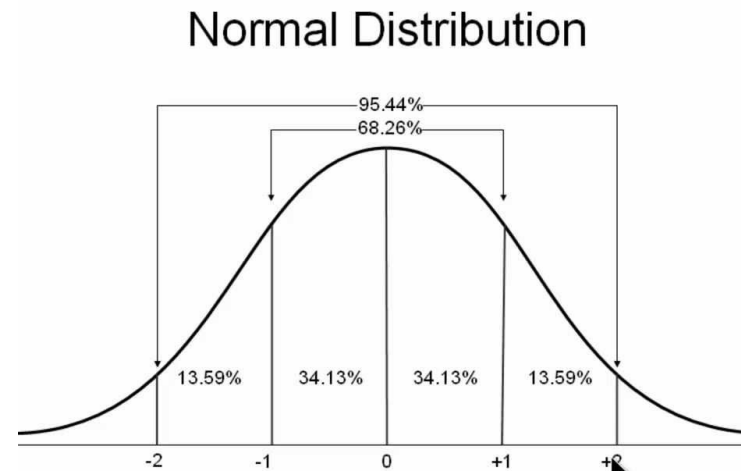
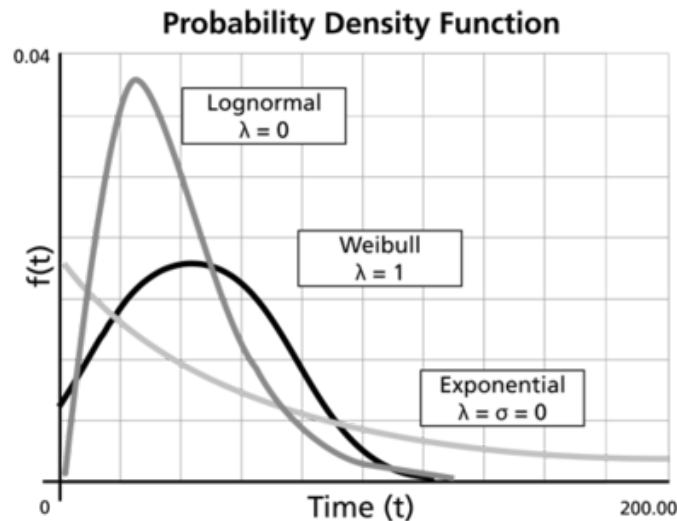
$$S(t) = S(t_0) + \int_{t_0}^t \Delta S(\tau) d\tau$$

Product lifetimes in the inflow-driven model

Lifetime data are scattered across the literature and often show large variations:

- NIES (Japan) lifetime database: <http://www.nies.go.jp/lifespan/>
- Consumer goods and vehicles: Müller et al. (2007): Service Lifetimes of Mineral End Uses. Report supported by U.S. Geological Survey (USGS), award number 06HQGR0174
- Best estimate for steel-containing products: DOI 10.1016/j.resconrec.2012.11.008 and DOI 10.1021/es303149z

Normal, Gamma, Exponential, and Weibull distributions are commonly applied to model product lifetimes.



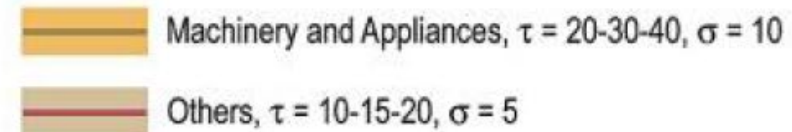
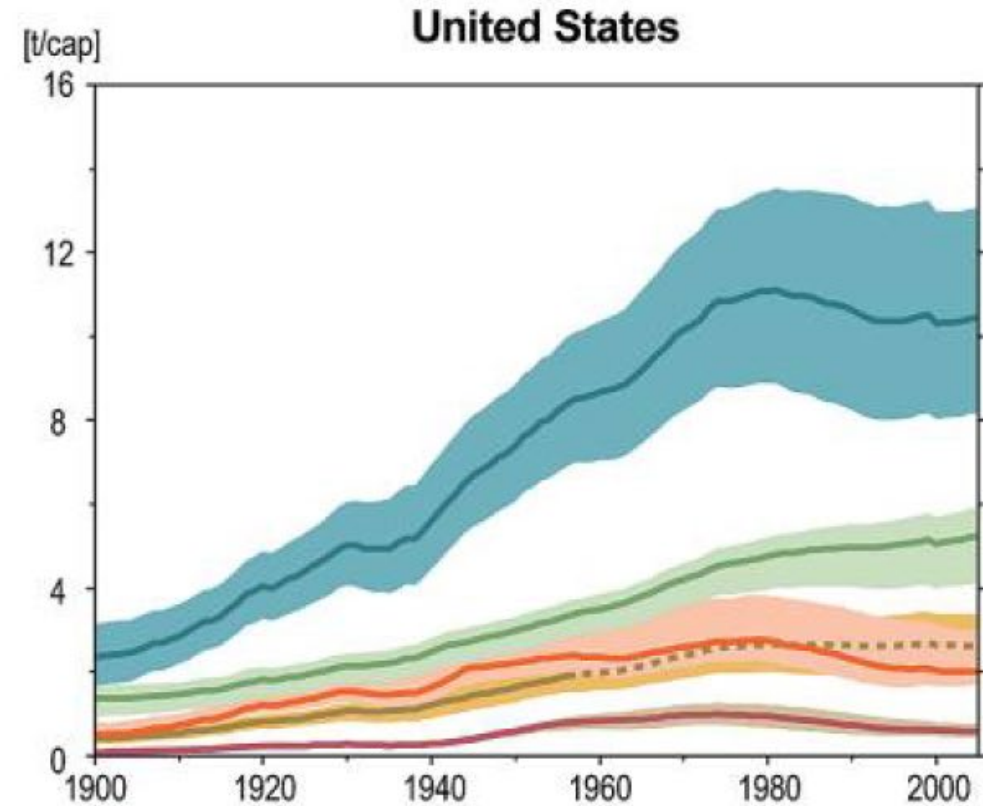
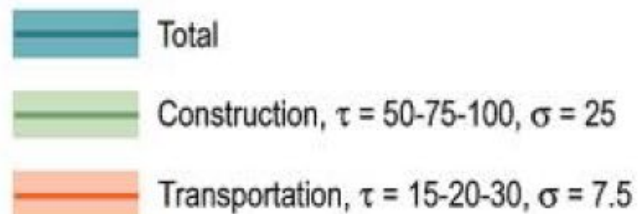
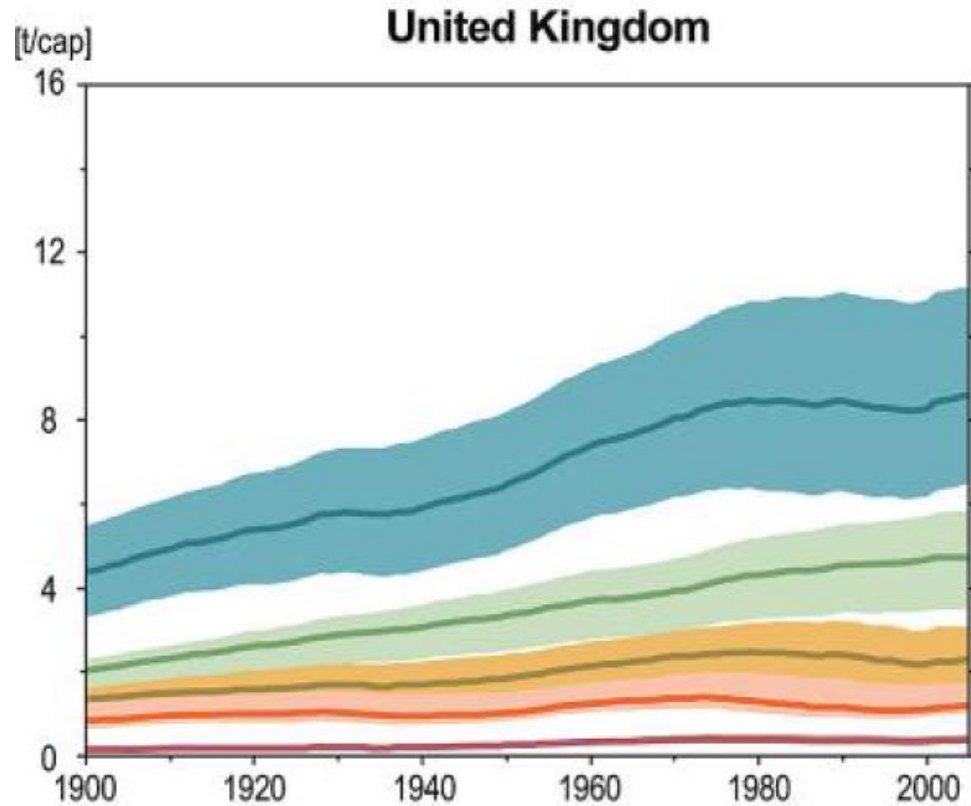
http://reliawiki.org/images/thumb/a/a5/WB_chp12_pdf.png/400px-WB_chp12_pdf.png

<https://i.ytimg.com/vi/xgQhefFOXrM/maxresdefault.jpg>

https://upload.wikimedia.org/wikipedia/commons/thumb/e/e6/Gamma_distribution_pdf.svg/325px-Gamma_distribution_pdf.svg.png

The inflow-driven model, example

Apparent saturation of per capita steel stocks in some industrialized countries



Dynamic MFA

Types of dynamic models III

The stock-driven model:

Introduction and software exercise

Transport - Vehicle Fleet - Transformation



The stock-driven model for determining inflows

To build scenarios for the future development of material cycles, one can

- Extrapolate or assume future consumption levels and then calculate the stocks and the services provided.
- Extrapolate or assume future service levels, infer the stocks required to deliver them, and calculate the inflows required to expand and maintain those stocks.

The latter approach is often more realistic, as it allows us to link the actual outcome of economic activity, service provision, directly to the socioeconomic variables providing those services, stocks.

The research question is then:

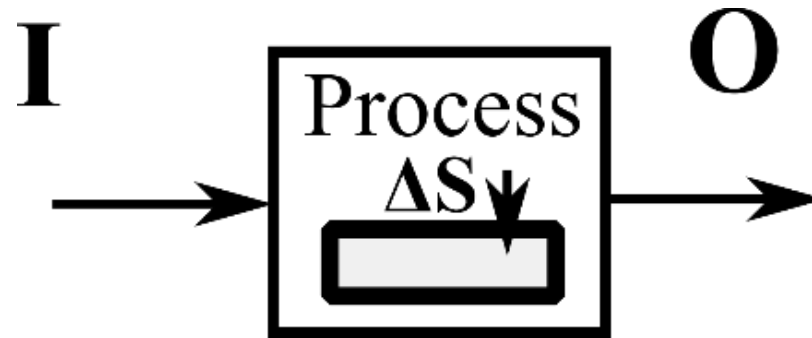
How large is the inflow needed to maintain and expand the in-use stock so that it fits a given scenario?

The method that answers this question is called stock-driven modelling.



Applying the stock-driven model to estimate in-use stocks

System definition:



Mathematically, the determination of an inflow from an outflow is the inverse of the convolution operation used for the inflow-driven model.

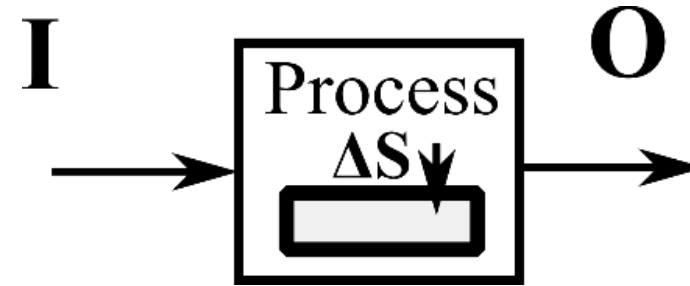
As the convolution involves the calculation of average values, its application to functions in general leads to a loss of information.

That means that the original signal (or inflow) can be reconstructed from the filtered signal (the outflow or stock) only in special cases.

In dynamic MFA such a special case is given if the age-cohort composition of the initial stock is known or that stock is zero.

Procedure of the stock-driven model to estimate in-use stocks

System definition:



Recursive procedure: Starting in the first model year, repeat the following steps for each year:

1) Calculate the outflow from the existing stock using the convolution of historic inflows:

$$O(t) = \int_{t_0}^t I(\tau) \cdot pdf(t - \tau) d\tau$$

2) Calculate the gap $\Delta S(t)$ between the actual stock and the remaining stock ($S_0 = 0$):

$$\Delta S(t) = S_{ext}(t) - S(t) = S_{ext}(t) - \int_{t_0}^t (I(\tau) - O(\tau)) d\tau = S_{ext}(t) - \int_{t_0}^t I(\tau) d\tau - \int_{t_0}^t \int_{t_0}^{\tau} I(\theta) \cdot pdf(\tau - \theta) d\theta$$

3) Set the inflow to fill the gap $\Delta S(t)$, where T is the time step of the model (e.g., 1 year):

$$I(t) = \Delta S(t) / T$$

Then repeat from step 1.

Exercise: a simple dynamic multi-layer stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

Year	Stock (cars)	Age-cohort						Use (km/yr)	Emissions (CO ₂ /yr)		
		-2	-1	0	1	2	3				
-2	4	4	0	0	0	0	0	10	240	$240 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 10 \text{ km/yr} * 6 \text{ CO}_2/\text{km}$	
-1	5	4	1	0	0	0	0	9	261	$261 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 9 \text{ km/yr} * 6 \text{ CO}_2/\text{km} + 1 \text{ car} * 9 \text{ km/yr} * 5 \text{ CO}_2/\text{km}$	
0	6							10			
1	7							10			
2	8							9			
3	9							9			
		Efficiency (CO ₂ /km)									
		6	5	5	4	4	3				

Exercise: a simple dynamic multi-layer stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

Year	Stock (cars)	Age-cohort						Use (km/yr)	Emissions (CO ₂ /yr)		
		-2	-1	0	1	2	3				
-2	4	4	0	0	0	0	0	10	240	$240 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 10 \text{ km/yr} * 6 \text{ CO}_2/\text{km}$	
-1	5	4	1	0	0	0	0	9	261	$261 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 9 \text{ km/yr} * 6 \text{ CO}_2/\text{km} + 1 \text{ car} * 9 \text{ km/yr} * 5 \text{ CO}_2/\text{km}$	
0	6	4	1	1	0	0	0	10			
1	7							10			
2	8							9			
3	9							9			
		Efficiency (CO ₂ /km)									
		6	5	5	4	4	3				

Exercise: a simple dynamic multi-layer stock model for the vehicle fleet

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		-2	-1	0	1	2	3				
-2	4	4	0	0	0	0	0	10	240	$240 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 10 \text{ km/yr} * 6 \text{ CO}_2/\text{km}$	
-1	5	4	1	0	0	0	0	9	261	$261 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 9 \text{ km/yr} * 6 \text{ CO}_2/\text{km} + 1 \text{ car} * 9 \text{ km/yr} * 5 \text{ CO}_2/\text{km}$	
0	6	4	1	1	0	0	0	10			
1	7	0	1	1	5	0	0	10			
2	8	0	0	1	5	2	0	9			
3	9	0	0	0	5	2	2	9			
		Efficiency (CO ₂ /km)									
		6	5	5	4	4	3				

Exercise: a simple dynamic multi-layer stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

Year	Stock (cars)	Age-cohort						Use (km/yr)	Emissions (CO ₂ /yr)		
		-2	-1	0	1	2	3				
-2	4	4	0	0	0	0	0	10	240	$240 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 10 \text{ km/yr} * 6 \text{ CO}_2/\text{km}$	
-1	5	4	1	0	0	0	0	9	261	$261 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 9 \text{ km/yr} * 6 \text{ CO}_2/\text{km} + 1 \text{ car} * 9 \text{ km/yr} * 5 \text{ CO}_2/\text{km}$	
0	6	4	1	1	0	0	0	10	340		
1	7	0	1	1	5	0	0	10			
2	8	0	0	1	5	2	0	9			
3	9	0	0	0	5	2	2	9			
		Efficiency (CO ₂ /km)									
		6	5	5	4	4	3				

Exercise: a simple dynamic multi-layer stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

Year	Stock (cars)	Age-cohort						Use (km/yr)	Emissions (CO ₂ /yr)		
		-2	-1	0	1	2	3				
-2	4	4	0	0	0	0	0	10	240	$240 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 10 \text{ km/yr} * 6 \text{ CO}_2/\text{km}$	
-1	5	4	1	0	0	0	0	9	261	$261 \text{ CO}_2/\text{yr} = 4 \text{ cars} * 9 \text{ km/yr} * 6 \text{ CO}_2/\text{km} + 1 \text{ car} * 9 \text{ km/yr} * 5 \text{ CO}_2/\text{km}$	
0	6	4	1	1	0	0	0	10	340		
1	7	0	1	1	5	0	0	10	300		
2	8	0	0	1	5	2	0	9	297		
3	9	0	0	0	5	2	2	9	306		
		Efficiency (CO ₂ /km)									
		6	5	5	4	4	3				

The stock-driven model for determining inflows

The stock-driven model is the inverse of the inflow driven model:

- The inflow computed by the stock-driven model is identical to the original inflow.
- The stock computed with the inflow-driven model is identical to the original stock.

Be creative with the initial stock!

- Use stock obtained from inflow-driven model to apply stock-driven model from a time when there were virtually no stocks.
- If the original stock age-cohort composition is unknown, the leaching model can be applied to S_0 .

Implementing the stock-driven model

In Excel: Possible but a bit cumbersome. See [IEooc Methods3 Software9](#) for an implementation.

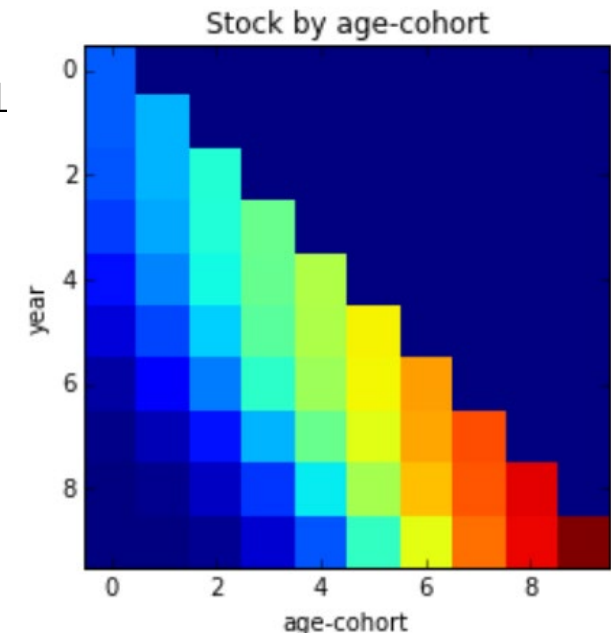
In Python: via Jupyter notebooks, e.g., [IEooc Methods3 Software1](#)

See section Methodology 3: Dynamic Material Flow Analysis of the IEooc

```
TestDSMX = DynamicStockModel(t = np.arange(1,11,1),  
s = np.array([ 2.5, 6. , 10.5, 16. , 22.5, 27.5, 32.5, 37.5, 42.5, 47.5]),  
lt = {'Type': 'Normal', 'Mean': np.array([4]), 'StdDev': np.array([1.0]) })
```

```
S_C, O_C, I, ExitFlag = TestDSMX.compute_stock_driven_model  
O, ExitFlag = TestDSMX.compute_outflow_total()  
DS, ExitFlag = TestDSMX.compute_stock_change()  
Bal, ExitFlag = TestDSMX.check_stock_balance()
```

→ **Applied in IEooc_Method3_Software1 (Python)**



Research & dissemination infrastructure @IEF



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Industrial ecology
open online course



Open source software:
ODYM MFA framework
RECC scenario model



General data model
and database:
Industrial ecology
data commons (iedc)

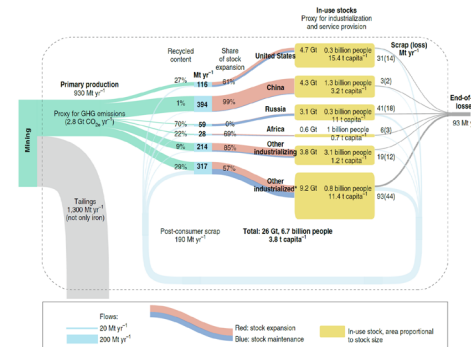
Guidelines and standards:
Methodology & indicators
Reproducibility of results
Traceability & provenance

Industrial Ecology Freiburg
WORKING PAPER

1
2023

Good Practice Examples for
Complete Traceability of
Workflows and Reproducibility
of Results in Industrial Ecology
Research

Interactive visualisation:
Sankey diagrams
Scenario results
Circular economy profiles



<https://www.industrialecology.uni-freiburg.de/>



**Thanks a lot
for your interest!**

www.industrialecology.uni-freiburg.de

Excursus: Application: Implications of low demand scenarios in the global building stock - materials and energy

Stefan PAULIUK¹, Fabio CARRER², Niko HEEREN², and Edgar G. HERTWICH²

1) University of Freiburg, 2) NTNU Trondheim, Norway

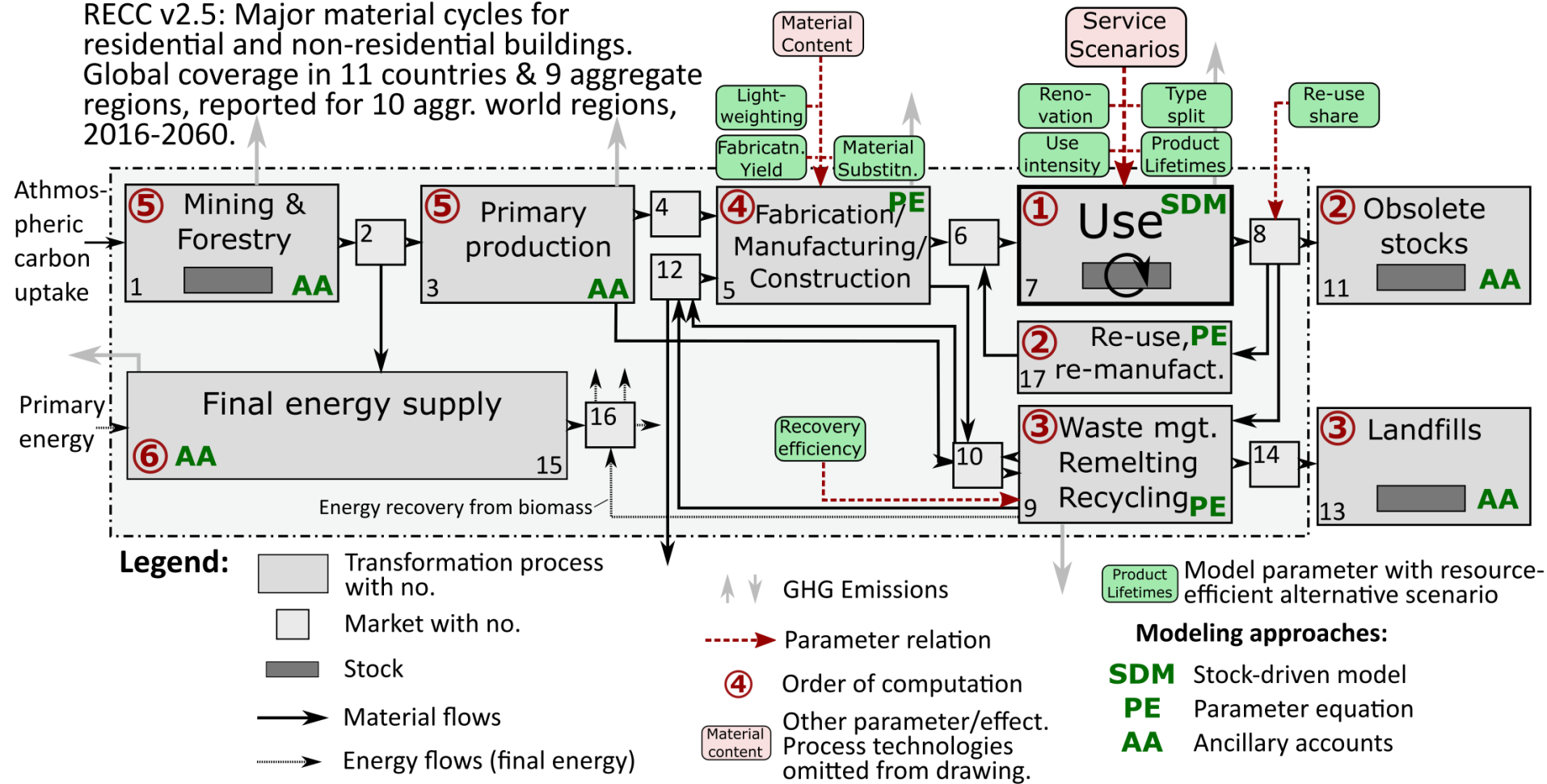
October 2024

Motivation

- Residential and non-residential buildings are a major contributor to human wellbeing.
 - Buildings cause 30% of final energy use, 18% of greenhouse gas emissions, and about 65% of material accumulation globally.
 - With electrification and higher energy efficiency, material-related emissions gain relevance.
 - The circular economy (CE) strategies, *narrow, slow, and close*, together with wooden buildings, can reduce material-related emissions.
- Comprehensive set of building stock transformation scenarios for ten world regions until 2060, using the RECC model of the stock-flow-service nexus, including low energy and material demand futures (LEMD: Values from Grubler et al. (2018) study)

System definition of the RECC model (resource efficiency and climate change mitigation)

RECC v2.5: Major material cycles for residential and non-residential buildings. Global coverage in 11 countries & 9 aggregate regions, reported for 10 aggr. world regions, 2016-2060.



Main driver: per capita floorspace for residential and non-res. buildings

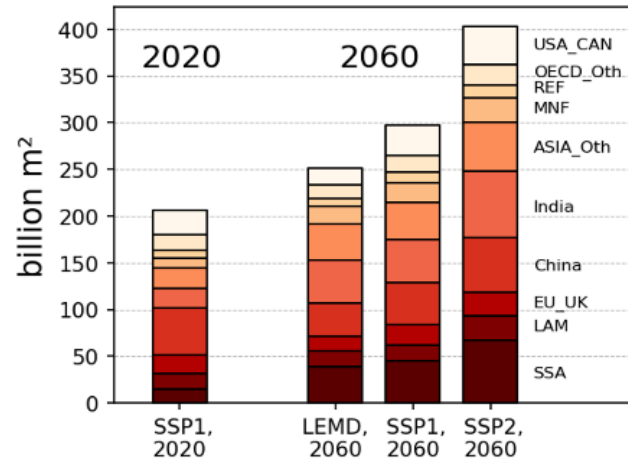
Table 1: Central parameters for the stock-flow-service nexus of the use phase: Initial and future service level (per capita floorspace) for the different socio-economic scenarios, and the typical building lifetime. Building lifetime can vary across age-cohorts and here, typical values are indicated. Region and scenario acronyms are defined in the text.

Regions	2015 per capita stock (m ²)		2050 per capita stock, m ² , LEMD / SSP1 / SSP2		Typical Building lifetime (yr)	
	residential	non-res.	residential	non-res.	residential	non-residential
SSA	11.4	0.8	19.4/26.9/33.4	7/10/12	50	45
LAM	34.4	3.0	30.3/34.4/44.3	7/10/12	50	45
EU_UK	37.7	12.5	31.2/40.1/46.2	12.8/16.1/20	100-180	60-80
China	36.1	10.8	31/40/50	13/16/20	27-40	30
India	11.7	0.8	25/28.7/38.1	7/10/12	50	45
Other_Asia	20.8	2.6	29.4/34.3/39	7.5/10.5/12.6	50	45
MNF	24.6	8.3	29.6/38.9/43.6	9/12/15	100	45
REF	23.5	5.9	29.5/38.9/43.5	9/12/15	120	60
Other_OECD	38.0	6.5	30.5/39.9/44.5	9/12/15	100	50
USA_CAN	66.8	24.1	42.5/66.8/83.7	18/26/30	110	45

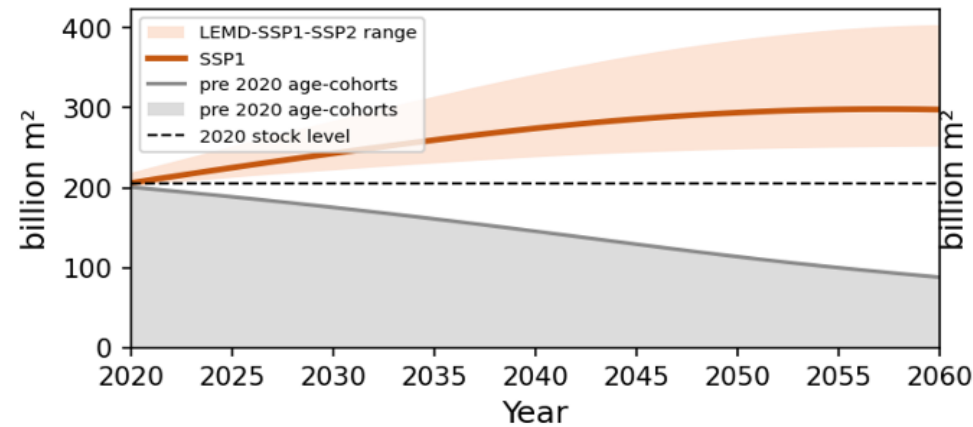
Stock and cumulative flows, global, 2020-2060

(a) Residential buildings, global

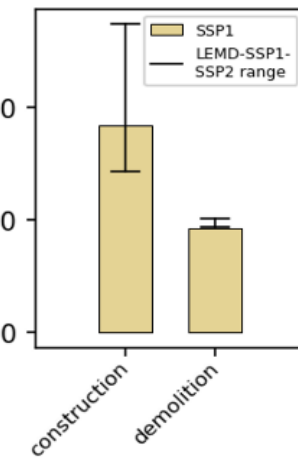
In-use stock by region, year, and scenario



stock over time

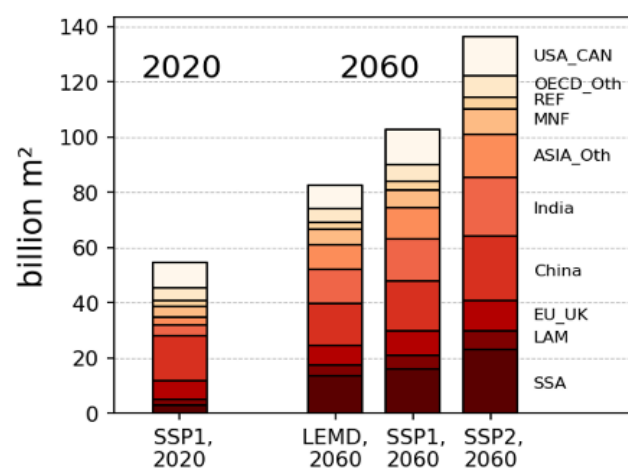


cumulative flows, 2020-2050

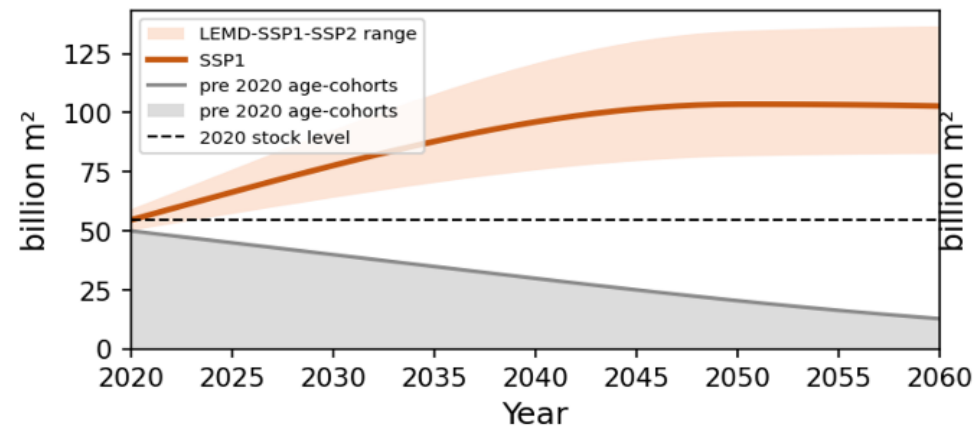


(b) Non-residential buildings, global

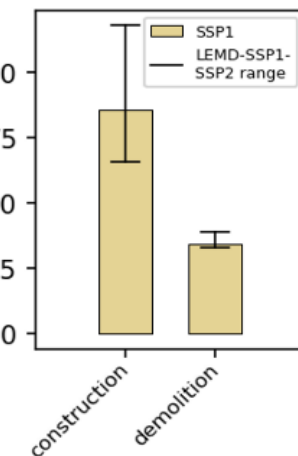
In-use stock by region, year, and scenario



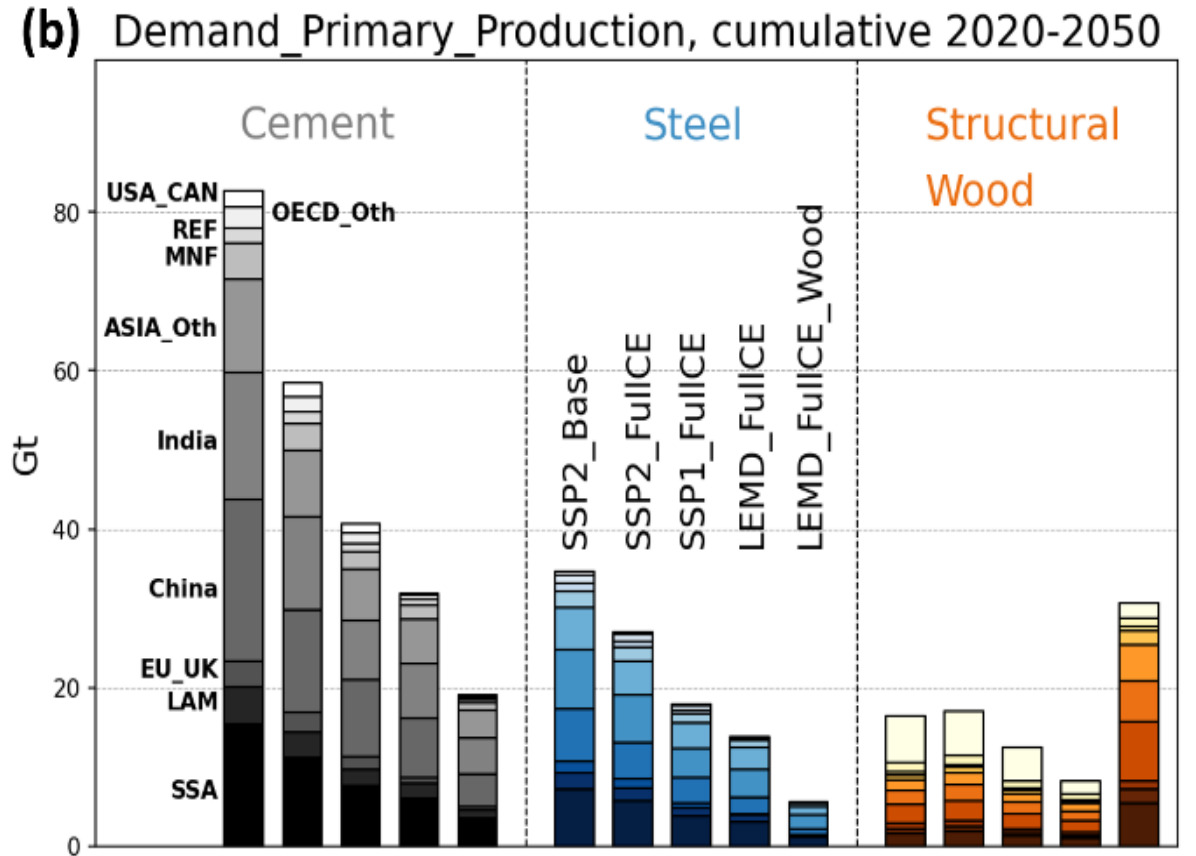
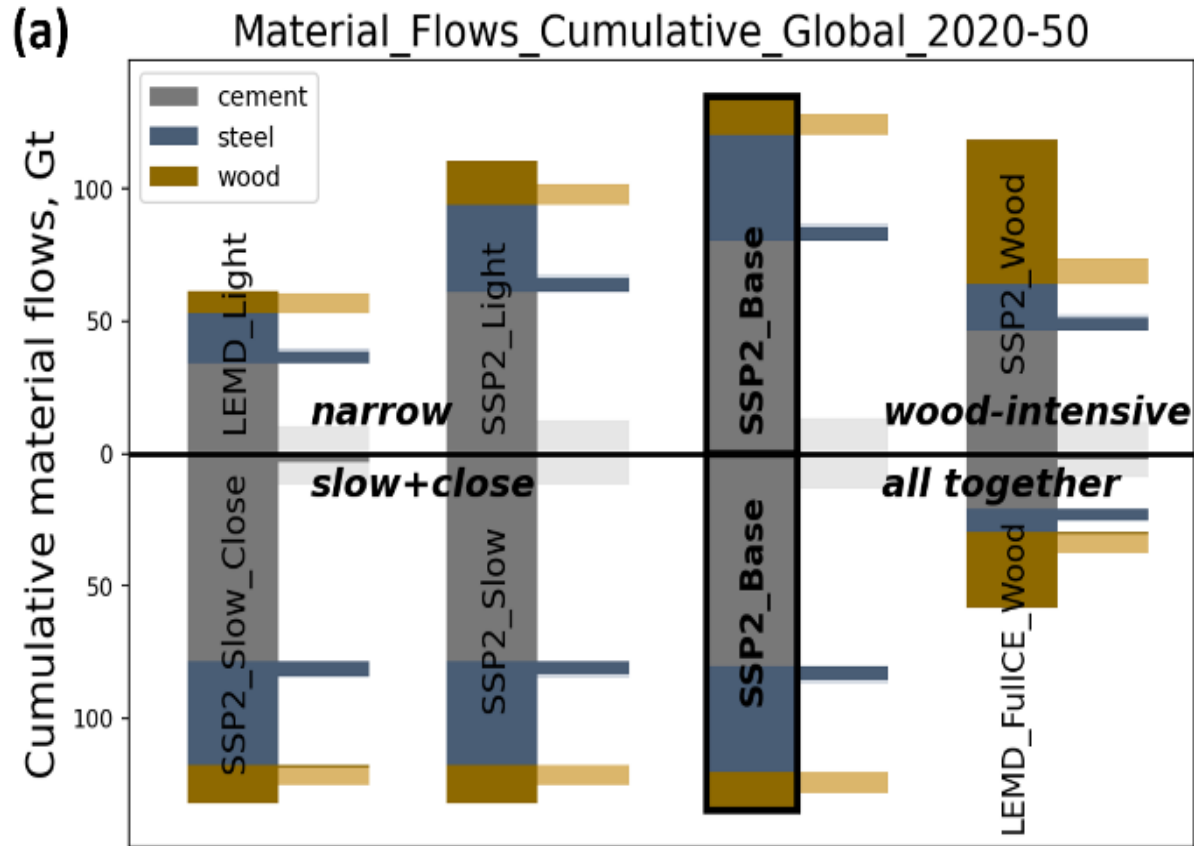
stock over time



cumulative flows, 2020-2050



Cumulative material flows, global, 2020-2060

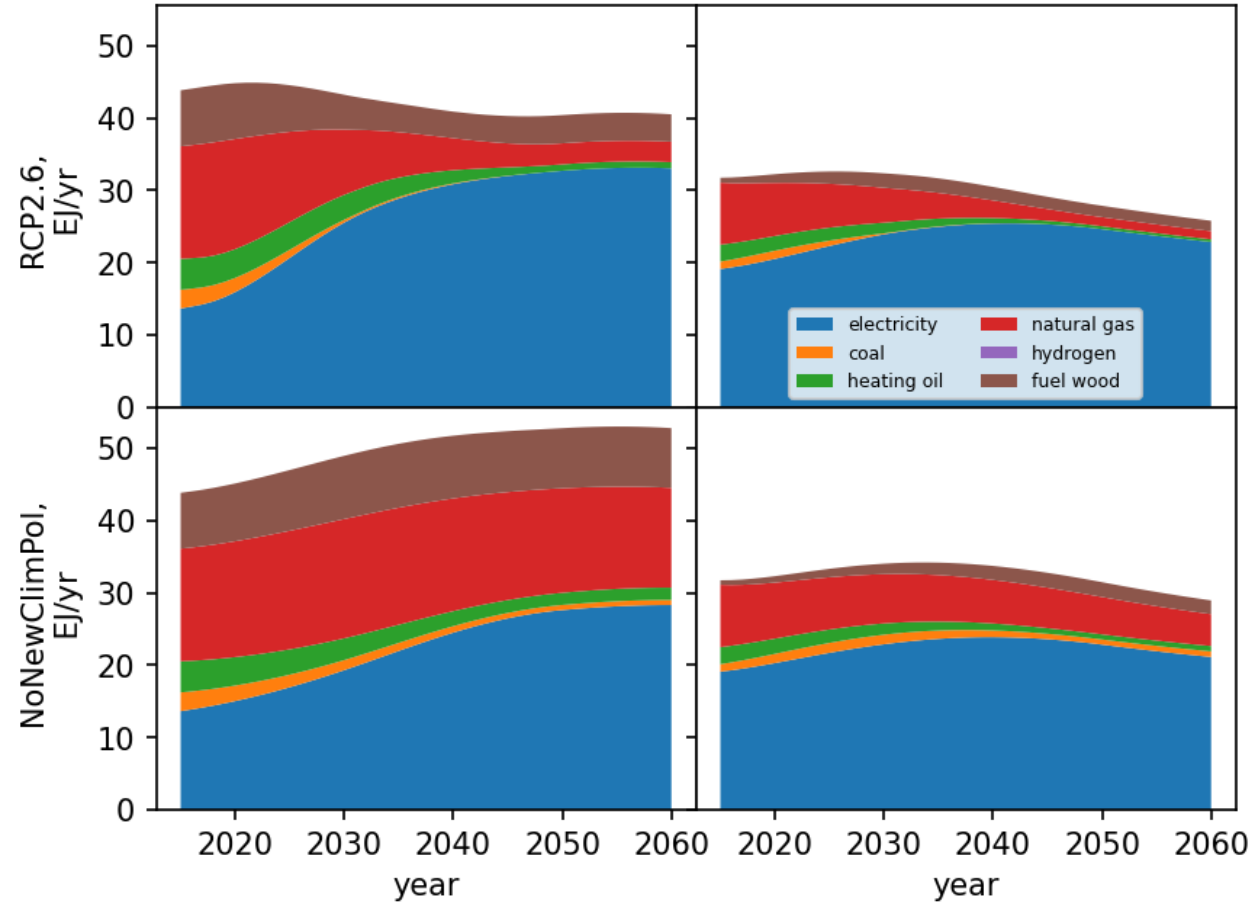


Use phase energy demand, global, 2020-2060

SSP2, energy demand, use phase, by scenario, Global

residential blds.

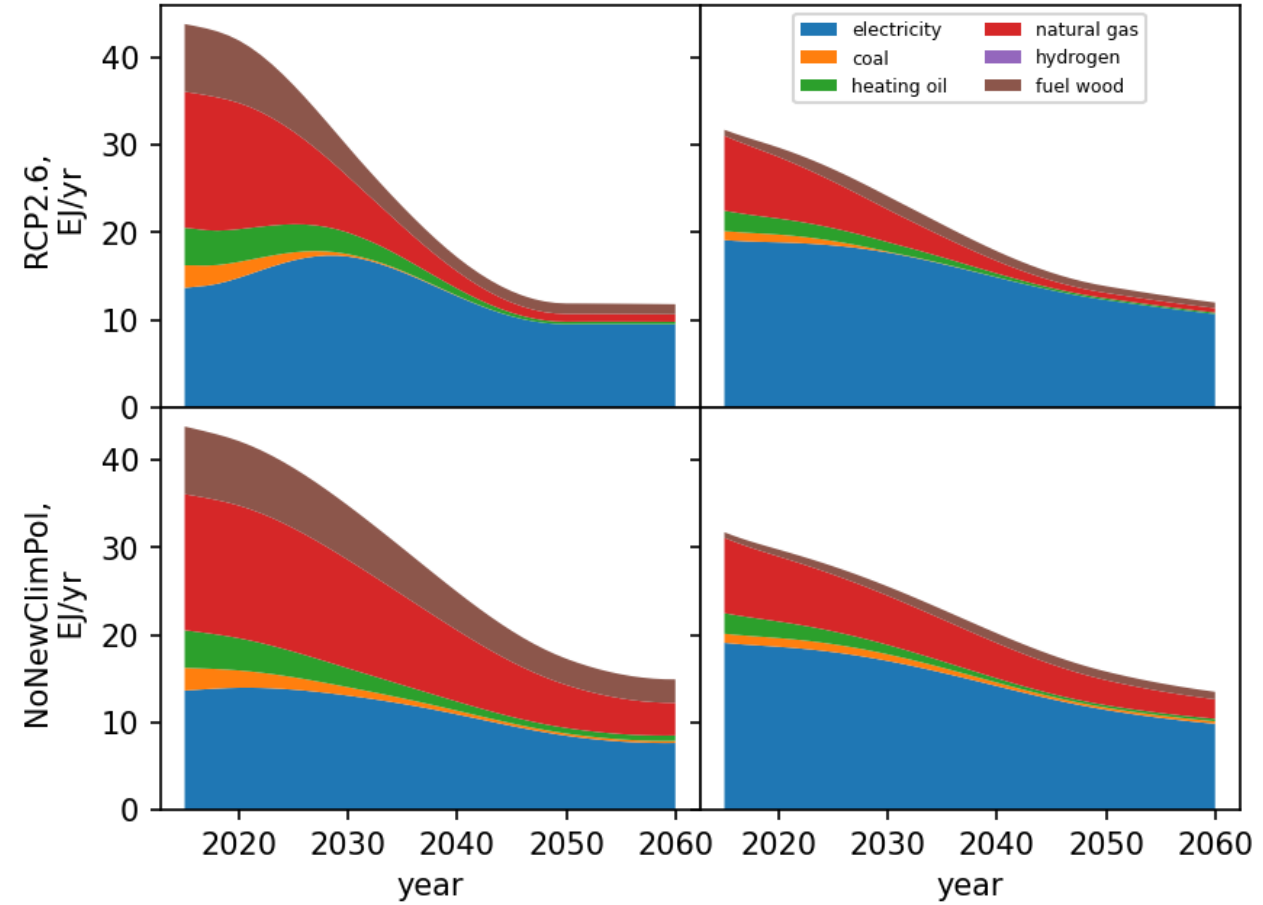
non-residential blds.



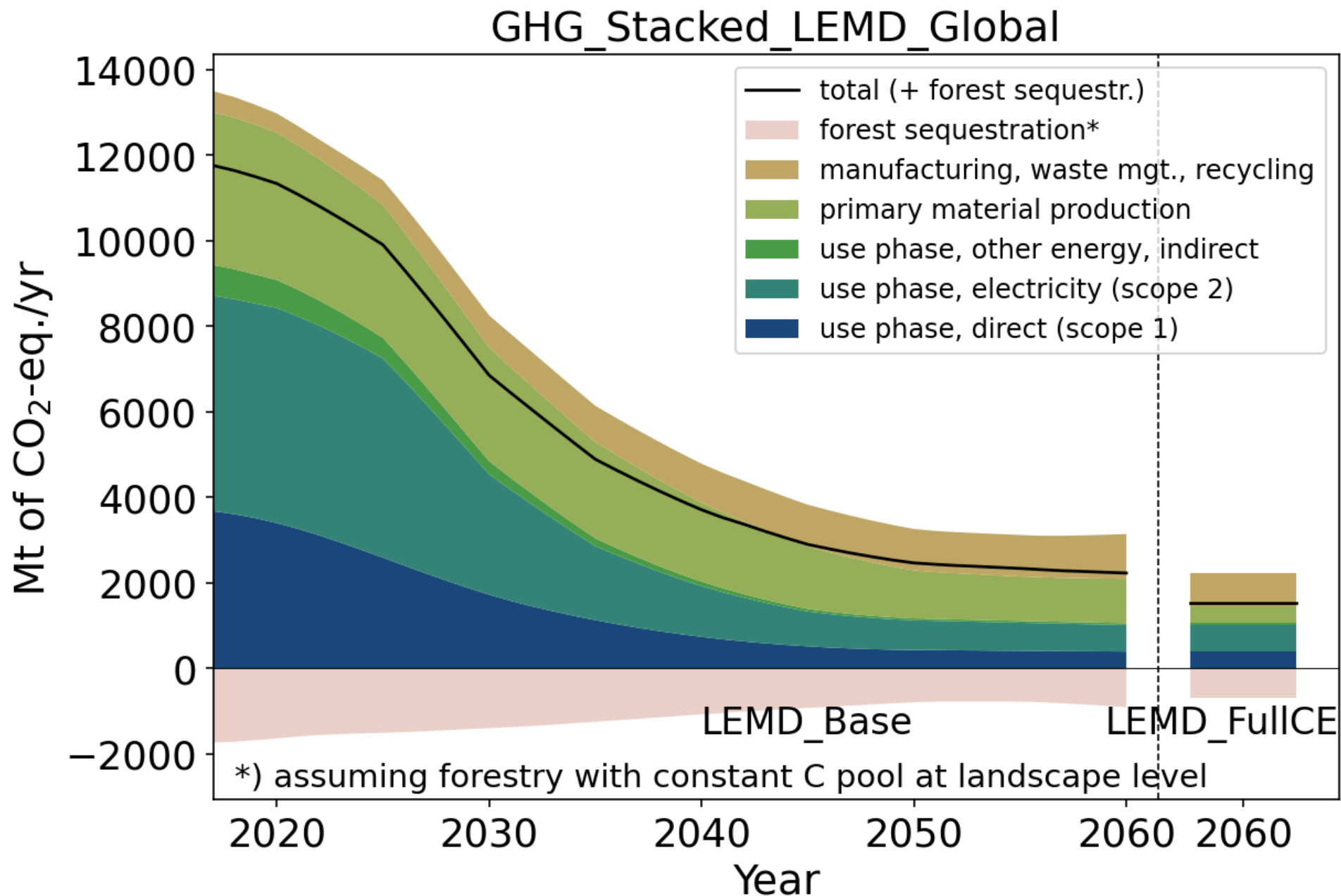
LEMD, energy demand, use phase, by scenario, Global

residential blds.

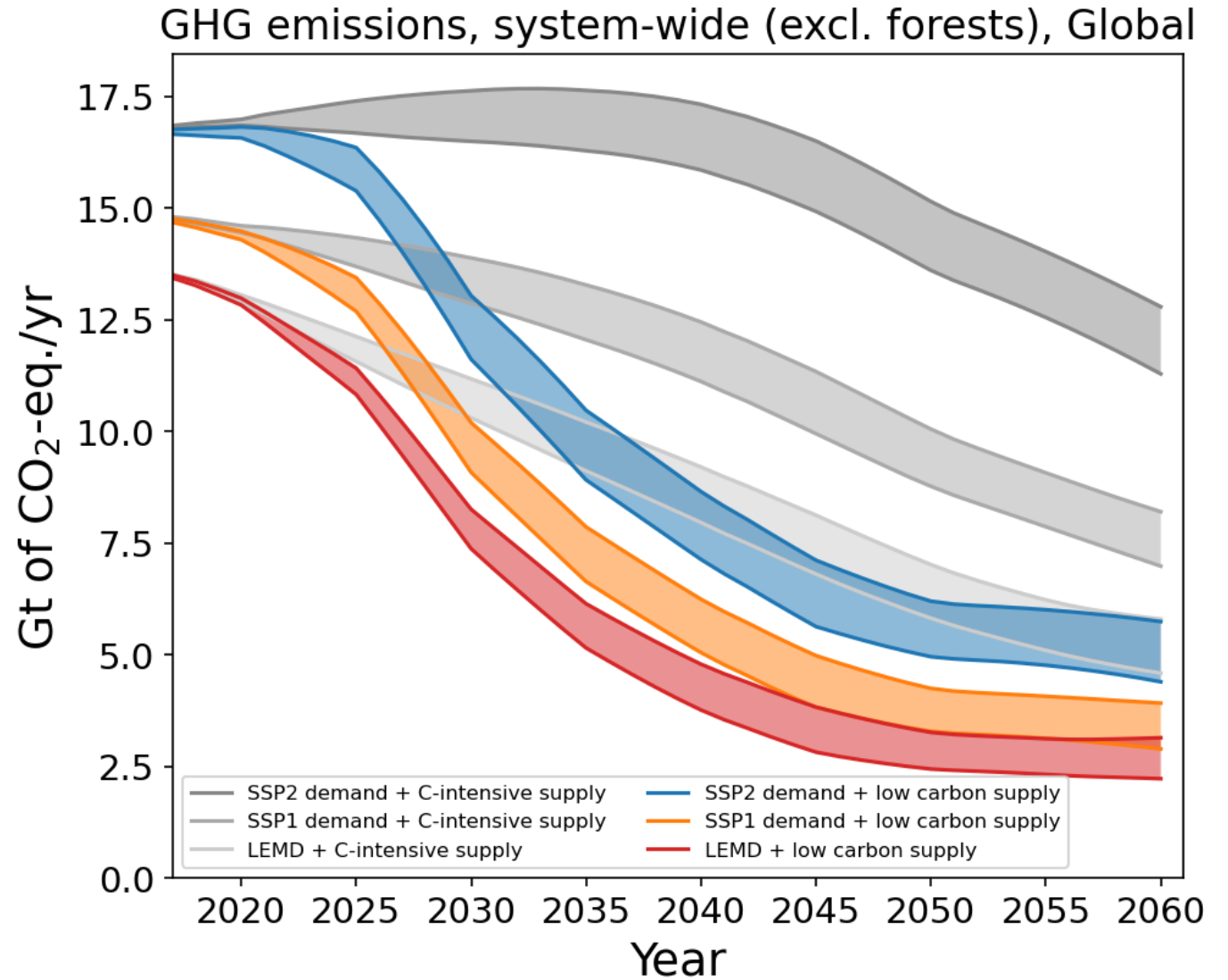
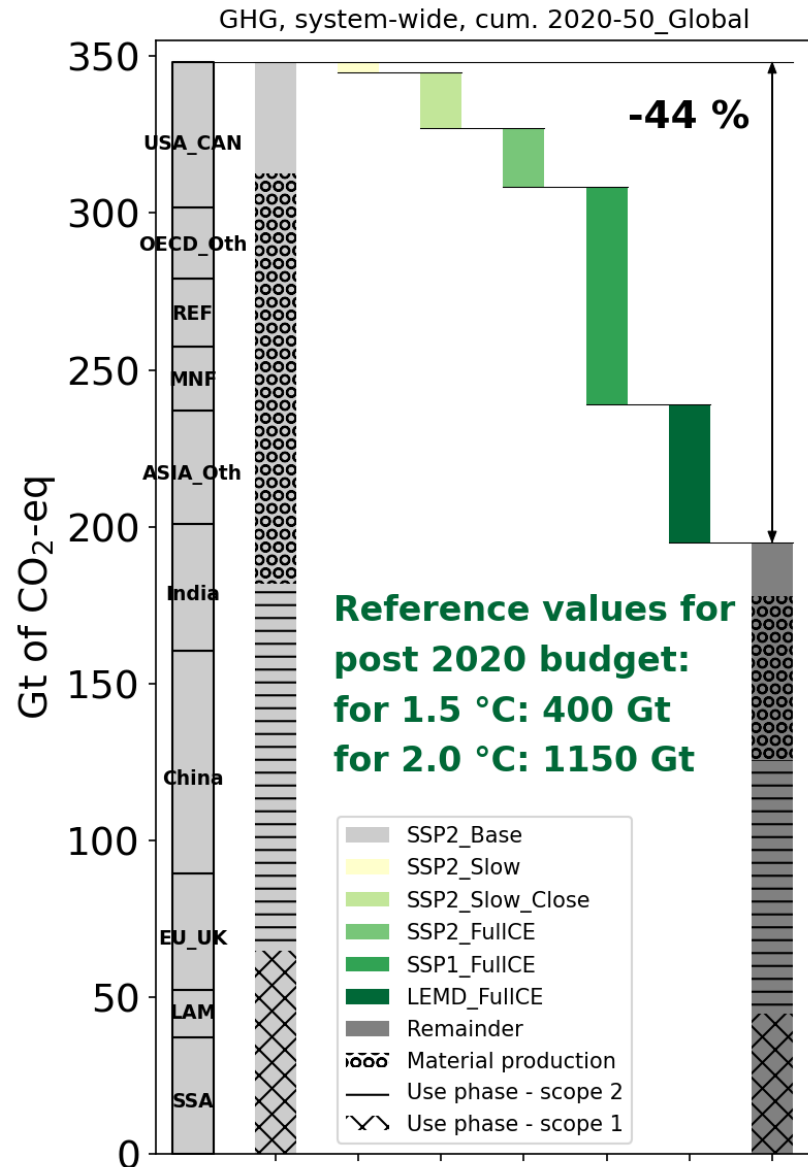
non-residential blds.



GHG time series by sector and region

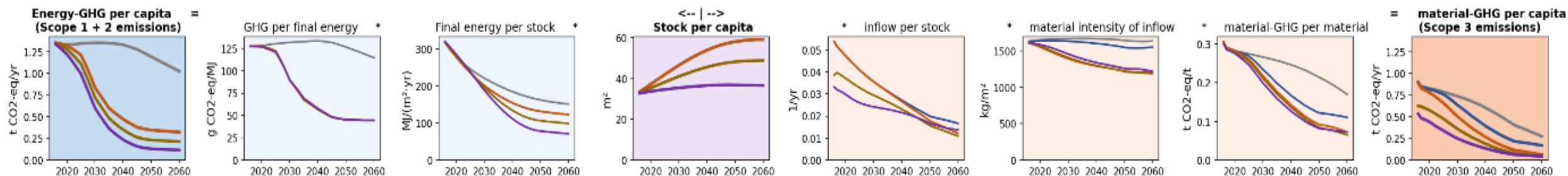


GHG reduction and scenario dependency

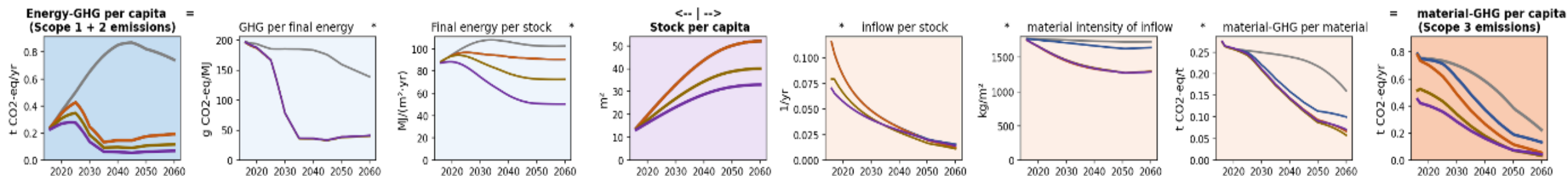


Decomposition analysis of Scope 1+2 and Scope 3 GHG

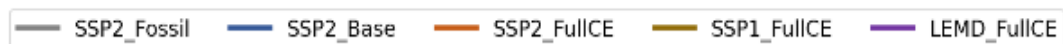
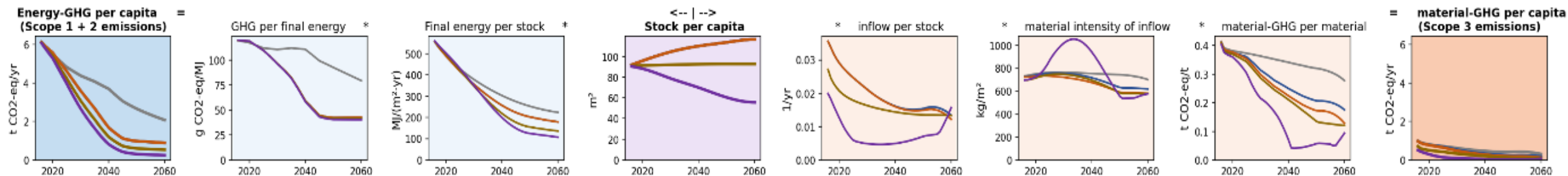
(a) Global aggregate



(b) India



(c) USA and Canada



Summary and outlook

- 2020-2050 global cumul. new construction ranges from 150 to 280 billion m² for residential and 70 120 billion m² for non-residential buildings. LEMD reduces cumul. 2020-2050 primary material demand from 80 to 30 gigatons (Gt) for cement and from 35 to 15 Gt for steel.
 - Lowering floor space demand by 1 m² per capita leads to global savings of 800-2500 megatons (Mt) of cement, 300-1000 Mt of steel, and 3-10 Gt CO₂-eq, depending on industry decarbonization and CE roll-out.
 - CE reduces 2020-2050 cumul. GHG by up to 44%, where the highest contribution comes from the narrow CE strategies, i.e., lower floorspace and lightweight buildings.
 - Very low carbon emissions trajectories are possible only when combining supply and demand-side strategies.
- Assess economic implications of LEMD! // Connect sector-level scenarios at the city scale!

Thank you for your attention!

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